Transportation Theory and Applications

Lecture I: Introduction

Fall 2017 - MTAT.08.043

A. Hadachi
Course Syllabus

Lecturer: Amnir Hadachi

Course website: https://courses.cs.ut.ee/2017/Transport/fall

Office hours: 12h - 14h Fridays

Schedule:
  Lectures: Th 12h15 - room 218
  Labs: Th 16h15 - room 219

Course materials:
Topics

Topics we will try to cover in this course:
1. Transportation Planning
2. Characteristics of transport problems
3. Data and spatial modelling
4. Trip generation models
5. Trip distribution
6. Modelling and decision making
7. Modal split and direct demand models
8. Transport modelling in practice
Grading

❖ Exercises and Homework (50 %)
❖ Final Exam (50 %)

❖ NB: To take the exam at least 60 % of Exercises and homework should be completed.
Transportation Theory

- Modelling transport is not done just for having fun with mathematical formulas, it has a purpose.
- Model is a simplified representation of the real world system and it is realistic from a particular perspective.
Transportation Theory

- Type of models:
  - Design models
    - (e.g. architecture, fluid mechanics, …).
  - Mental models
    - (Used in our daily life to help understand and interpreting the real world and the abstract model).
    - Enhanced via discussion, training and experience.
  - Abstract models
    - Based on mathematical model.
Characteristics of transport problems

- Factors:
  - Congestions, delays, accidents, environment

\[
\begin{aligned}
\sum_{i=1}^{p} O_{ij} &= \sum_{j=1}^{m} D_{ij} \quad \text{Balanced} \\
\sum_{i=1}^{p} O_{ij} &\neq \sum_{j=1}^{m} D_{ij} \quad \text{Unbalanced}
\end{aligned}
\]
❖ Traffic flow
  - Urban planning
  - General assessment
  - Management

❖ Traffic information
  - Travel time information
  - Forecasting
  - Traffic jams
  - Incident detection

❖ Network status and performance
  - Modality
  - Number of vehicles
Why

❖ You need to take a decision on:
  ❖ Building new roads
  ❖ Maintaining the infrastructure
  ❖ Modifying the public transport
  ❖ Upgrading the circulation in the city
  ❖ Modifying policies
❖ The need to understand a situation or assess its impact
  ❖ Pollution
  ❖ Jams
  ❖ Accessibility
  ❖ Economy
Figure 8. Mobility savings framework

Why
Transportation Theory

- The main goal behind transportation theory is to model transport in order to provide a quantitative information to make our mobility smarter with respect to our environment.

Key Tool for Decision Making
Transportation Planning

❖ Process:

- Problem
- Aims

Stage:

- Assessment
- Solution
- Analysis & Forecasting
- Evaluation

Decision

modelling
Transportation Planning

- Process:

  - Evaluation
  - Current status
  - Future perspectives
  - Planning
  - Environment
  - Infrastructure
  - Public Transport
The model

How to do proper representation?
The model

Tips:

- Define the main mechanisms, their decomposition and relationships between its components.

Challenges

- Non-trivial
- Non-linear
- Needs many feedback
Transportation mechanisms

Activities → Transport Mode → Accessibility → Land/Build/Space
Transportation mechanisms

Activities
- Trip
- Ability to travel
- Move
- Choice location

Accessibility
- Destination
- Route/time
- Travel time
- Cost
- Attractiveness
- Location / investors

Land/Build/Space
- Transport Mode
transport model

Zones networks → Base year data

Database
  Base Year
  Future

Trip generation → Distribution

Modal split → Assignment

Evaluation → output

Future planing data
Theoretical approach

❖ Descriptive models
  ❖ (Based on statistics or analogy to find out the relationship between input and output.)

❖ Models based on choice theory
  ❖ (solving a maximisation problem, or minimisation)

❖ Equilibrium concept
Empirical approach

❖ Focus on:
  ❖ Nature of data
  ❖ Build a model based on data
  ❖ Test the model based on data
Data

❖ Nature of data
  ❖ Transport system
    ❖ Road network
    ❖ public transport
    ❖ policies
  ❖ Semantic information
    ❖ population
    ❖ locations of interest
    ❖ shops
  ❖ Environment
Choice Theory

- Utility maximisation/minimisation
- Travel choice theory
Utility maximisation
Utility maximisation

Constraints

maximising personal utility

minimisation of travel time is not the only concern
Minimisation

- In general:
  - finding the minimum cost of transporting a single commodity from A to B locations.
  - Usually can be solved just by network methods (e.g. finding the shortest path)
Minimisation

Example:

\[ x_i \]: quantity of the commodity transported

\[ c_i \]: the cost of transporting
Minimisation

- Example:
  - The cost associated with this mobility is:
    - cost * quantity
  - Thus,
    
    \[ Total \ cost = \sum_{i=1}^{p} \sum_{j=1}^{m} c_{ij} x_{ij} \]
Minimisation

Example:

Summary: in order to minimise the transportation cost we have to solve the following:

- minimize:  
  \[ z = \sum_{i=1}^{p} \sum_{j=1}^{m} c_{ij}x_{ij} \]

- subject to:  
  \[ \sum_{j=1}^{m} x_{ij} \leq O_i \text{ for } i = 1, \ldots, p \]

- and  
  \[ \sum_{i=1}^{p} c_{ij} \geq D_j \text{ for } j = 1, \ldots, m \]

- where:  
  \[ x_{ij} \geq 0 \text{ for all } i \text{ and } j \]
Balanced transportation model

<table>
<thead>
<tr>
<th></th>
<th>storage 1</th>
<th>storage 2</th>
<th>storage 3</th>
<th>supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factory 1</td>
<td>$c_{11}$</td>
<td>$c_{12}$</td>
<td>$c_{13}$</td>
<td>20</td>
</tr>
<tr>
<td>Factory 2</td>
<td>$c_{21}$</td>
<td>$c_{22}$</td>
<td>$c_{23}$</td>
<td>10</td>
</tr>
<tr>
<td>Demand</td>
<td>7</td>
<td>10</td>
<td>13</td>
<td></td>
</tr>
</tbody>
</table>

*Balanced transportation model*

**Example 1.1**

Total supply?
Total demand?
### Balanced transportation model

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</tbody>
</table>

Demand:
- 7
- 10
- 13

Total supply $= 20 + 10 = 30$

Total demand $= 7 + 10 + 13 = 30$

**Example 1.1**

Total supply $=$ Total demand
Solving the transportation problem

❖ recap:
\[
\sum_{i=1}^{p} O_{ij} = \sum_{j=1}^{m} D_{ij}
\]

❖ Finding feasible solution
  ❖ [1. North-west Corner Method]
  ❖ [2. Least Cost Method]
Solving the transportation problem

❖ 1. North-west Corner Method

❖ Steps

A. Start from the north-west corner box in the table by allocating the maximum amount allowable by the supply and demand constraints to the variable \( x_{11} \).

B. Exit the row or the column when the supply or demand reaches zero and cross it out, to show that you cannot make any more allocations to that row or column.

C. If a row or a column simultaneously reach zero, only cross out one (the row or the column) and leave a zero supply (demand) in the row (column) that is not crossed out.

D. Continue the process till only one row or column is left. Then, you can compute the cost of transportation.
Solving the transportation problem

- **Example**

<table>
<thead>
<tr>
<th></th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>F1</strong></td>
<td>5</td>
<td>1</td>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td><strong>F2</strong></td>
<td>9</td>
<td>4</td>
<td>0</td>
<td>14</td>
</tr>
<tr>
<td><strong>F3</strong></td>
<td>17</td>
<td>6</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td><strong>Demand</strong></td>
<td>9</td>
<td>10</td>
<td>11</td>
<td></td>
</tr>
</tbody>
</table>
Solving the transportation problem

❖ 1. Least-Cost Method

❖ Steps

A. Assign to the cell with the smallest unit cost as much as possible. In case there is a tie then choose arbitrarily.

B. when a row or column satisfy the supply or demand cross it. In case a column or row are both satisfied then cross out only one of them.

C. continue the process till one row or column is left. Then, compute the cost.
Solving the transportation problem

**Example**

<table>
<thead>
<tr>
<th></th>
<th>S1</th>
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<td>11</td>
<td></td>
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</table>
Checking optimal solution

❖ So far we found initial basic solution
❖ is it really the optimal solution?
❖ For this reason we will introduce MODI method or U-V method (Modified distribution method) is an optimisation techniques used to find the optimal transportation cost.
❖ In our case we will modify the initial basic solution in order to find the optimal one.
Checking optimal solution

- Let’s suppose the cost $c_{ij}$ has two component dispatch cost $u_i$ and reception cost $v_j$, Thus: $c_{ij} = v_j + u_i$
- The total number of basic variable are $m+p-1$
- The total number of $m+p$
- Hence we can assign 0 to $u_1 = 0$
Checking optimal solution

- and we will use $P_{ij}$ to reflect the penalty regarding the cost since:
  $$P_{ij} = c_{ij} - (u_i + v_j)$$

- Process:
  - we assign values of $u_i$ and $v_j$ to the columns.
  - we enter the values $P_{ij} = c_{ij} - (u_i + v_j)$ in every cell.
  - if all the $P_{ij}$'s are negative, we have an optimal solution.
### Unbalanced transportation model

#### Example 1.2

<table>
<thead>
<tr>
<th>storage 1</th>
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<tbody>
<tr>
<td><strong>Factory 1</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c&lt;sub&gt;11&lt;/sub&gt;</td>
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<td>c&lt;sub&gt;13&lt;/sub&gt;</td>
<td>20</td>
</tr>
<tr>
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<td></td>
<td></td>
<td></td>
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<td>c&lt;sub&gt;23&lt;/sub&gt;</td>
<td>10</td>
</tr>
</tbody>
</table>

| Demand | 9 | 10 | 13 |

The model is unbalanced because

\[
\sum_{i=1}^{p} O_{ij} \neq \sum_{j=1}^{m} D_{ij}
\]
Example 1.2

❖ Tow case:

❖ demand exceeds the supply
❖ or
❖ supply exceeds the demand
### Unbalanced transportation model

**Example 1.2**

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<td>$c_{21}$</td>
<td>$c_{22}$</td>
<td>$c_{23}$</td>
<td>10</td>
</tr>
<tr>
<td>imaginary factory</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Demand</td>
<td>9</td>
<td>10</td>
<td>13</td>
<td></td>
</tr>
</tbody>
</table>

Demand exceeds the supply
Example 1.2

- Situations to consider:
  - the transportation costs can be considered as zero
  - or
  - Penalty cost “P”
### Example 1.2

A unbalanced transportation model is demonstrated below. The model consists of three storage units connected to two factories and a demand, where the supply exceeds the demand.

<table>
<thead>
<tr>
<th>Storage Units</th>
<th>Factory 1</th>
<th>Factory 2</th>
<th>Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$c_{11}$</td>
<td>$c_{21}$</td>
<td>7</td>
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<td></td>
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<td>$c_{22}$</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>$c_{13}$</td>
<td>$c_{23}$</td>
<td>13</td>
</tr>
<tr>
<td>Storage 3</td>
<td>0</td>
<td>0</td>
<td>4</td>
</tr>
</tbody>
</table>

Supply:
- Factory 1: 20 units
- Factory 2: 14 units

Demand:
- Total: 4 units

Supply exceeds the demand.
Example 1.2

- Situations to consider:
  - the transportation costs can be considered as zero
  - or
  - Storing cost “S”
Travel choice theory

- Travel behaviour
  - vary from one person to another
  - waiting times are usually overestimated
  - cost of driving a car or taking public transport may seem less expensive

All depends on the travel's perception