General description

Jacobi method

Solve $Ax = b$ using Jacobi method – find x iteratively starting at initial guess $x^{(0)}$ and the following formula to calculate each next approximation $x^{(k)}$, $k = 1, \ldots$

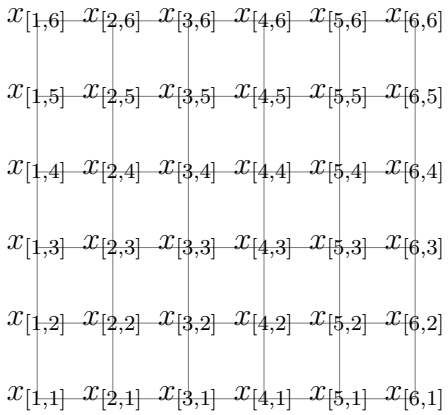
$$x_i^{(k)} = \frac{1}{a_{ii}} \left(b_i - \sum_{j \neq i} a_{ij} x_j^{(k-1)} \right), \quad i = 1, \ldots, n$$

Our task

In our case

$$A = \begin{pmatrix} 4 & -1 & \cdots & 0 \\ -1 & 4 & & \vdots \\ \vdots & & \ddots & -1 \\ 0 & \cdots & -1 & 4 \end{pmatrix}, \quad x = \begin{pmatrix} x_{[1,1]} \\ x_{[1,2]} \\ \vdots \\ x_{[6,6]} \end{pmatrix}, \quad b = \begin{pmatrix} b_{[1,1]} \\ b_{[1,2]} \\ \vdots \\ b_{[6,6]} \end{pmatrix}$$

have strange indexing because we get the unknowns $x_{[i,j]}$ from the grid

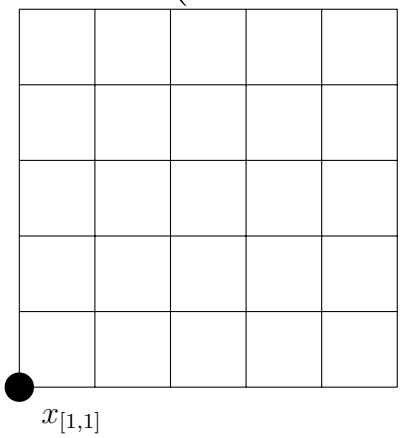


where

- $x_{[i,j]}$ – unknown quantity we are looking for (temperature, surface height, electric charge) in node (i, j)
- $b_{[i,j]}$ – external "forces" (heat source, gravity) in node (i, j)
- A – reflects the relation between the quantities in neighbor nodes and the "forces" in the node

Instructions

your id (location in grid): 1,1



Your $b_{[1,1]} = 0$

You are responsible for calculating $x^{(k)}_{[1,1]}$ element and should ask your neighbours for their x values from the previous step $(k-1)$

$$x^{(k)}_{[1,1]} = \frac{b_{[1,1]} + x^{(k-1)}_{[0,1]} + x^{(k-1)}_{[2,1]} + x^{(k-1)}_{[1,0]} + x^{(k-1)}_{[1,2]}}{4}$$

If index (i, j) of $x^{(k-1)}_{[i,j]}$is out of bounds, assume $x^{(k-1)}_{[i,j]} = 0$.

Compute:

$x^{(0)}_{[1,1]} = 0$

$x^{(1)}_{[1,1]} =$_____

$x^{(2)}_{[1,1]} =$_____

$x^{(3)}_{[1,1]} =$_____

$x^{(4)}_{[1,1]} =$_____

$x^{(5)}_{[1,1]} =$_____