Introduction to Optimization Methods

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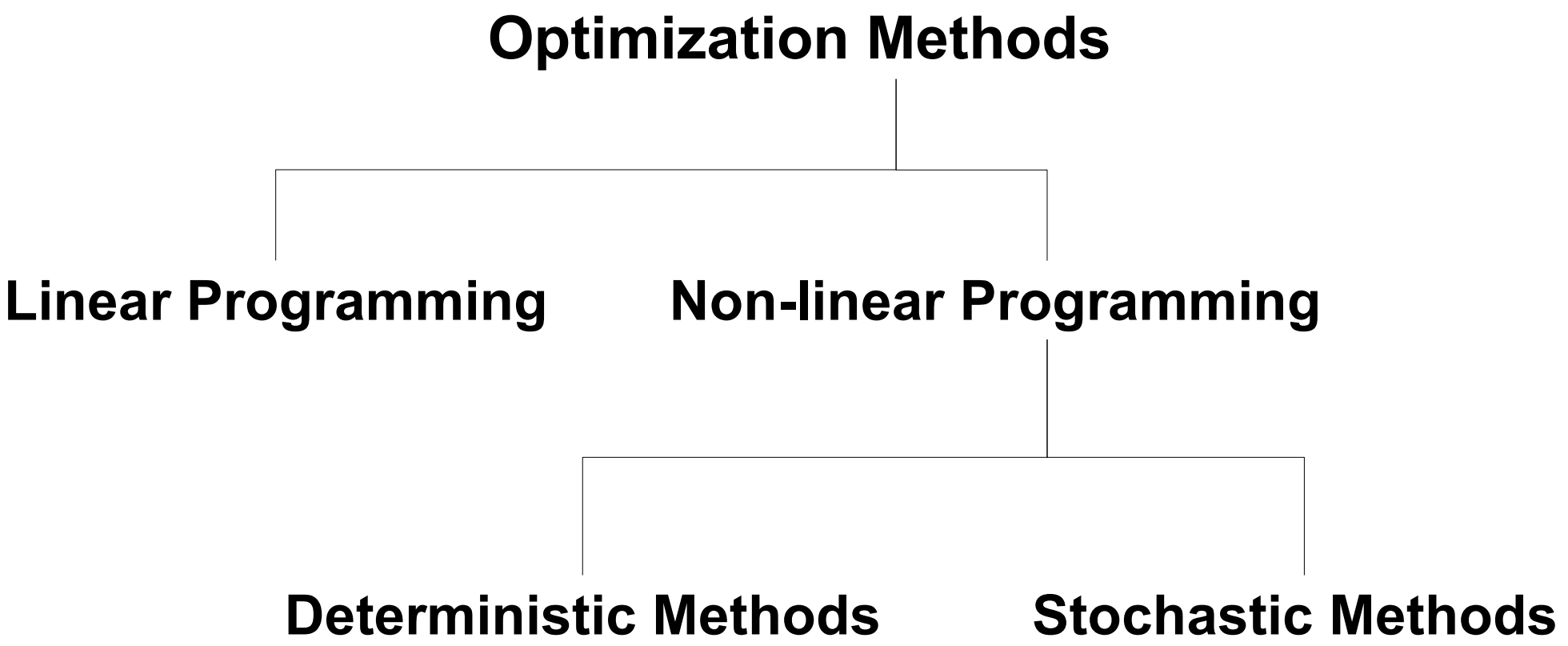
What is Optimization?

- Finding the *best solution*.
- Minimize cost and/or maximize performance.


Optimization Methods
Linear Programming
Non-linear Programming
Deterministic Methods
Stochastic Methods

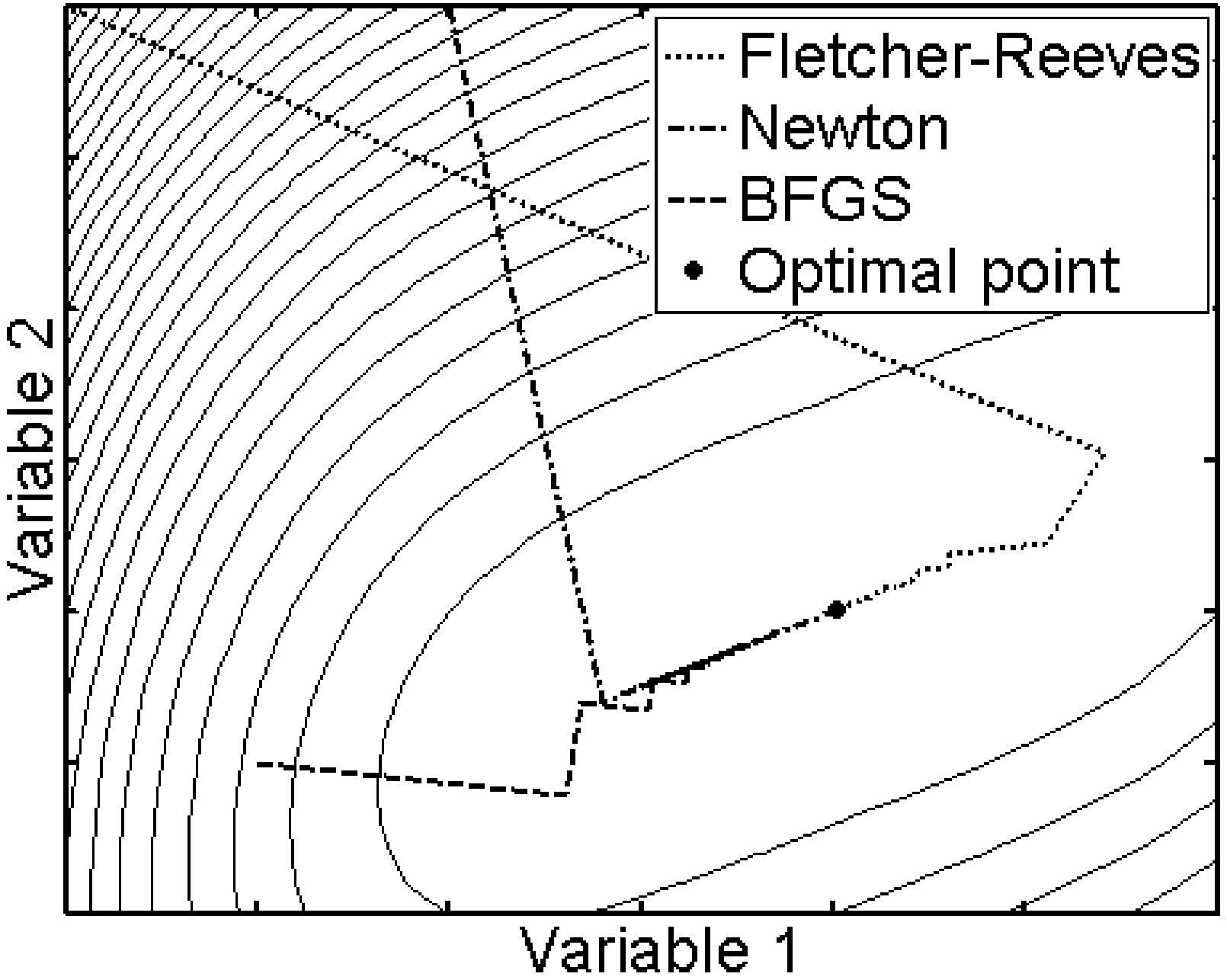
Linear Programming

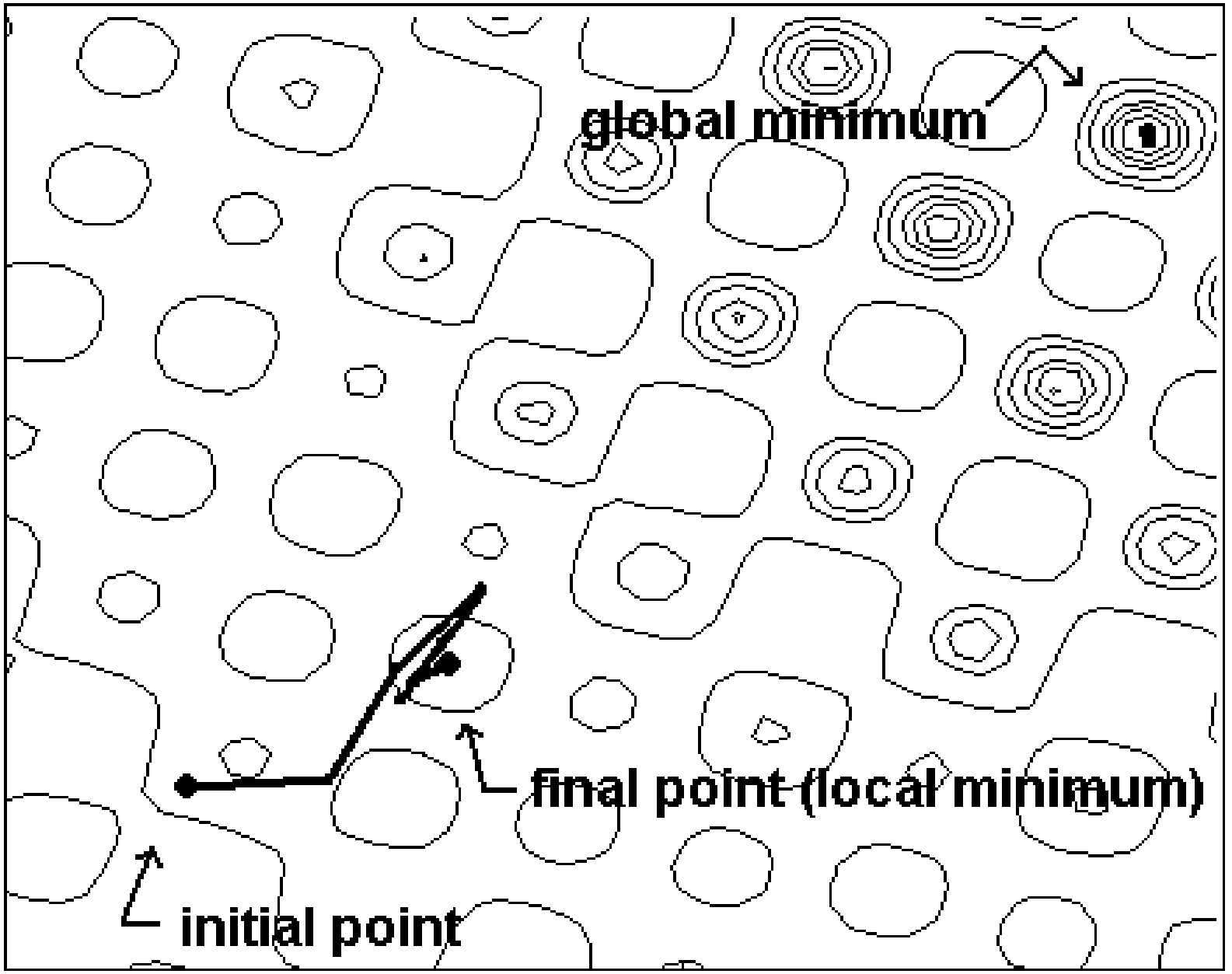
- Objective functions – functions that need to be maximized or minimized.
- Restrictions.
- Optimal solutions is the one that minimizes (or maximizes) the objective function.
- Traveling Salesman Problem (TSP)

Non-linear Programming

- Deterministic Methods:
  - Methods for finding direction (Newton's method, Fletcher-Reeves, **BFGS**)
  - Methods for finding step length (Golden Section, Fibonacci)
  - Yield good results only with functions that are **continuous, convex** and **unimodal**.

$$f(x, y) = (x - 2)^4 + (x - 2y)^2$$


Fletcher-Reeves
Newton
BFGS
Optimal point
Variable 2
Variable 1


global minimum
final point (local minimum)
initial point

Stochastic Methods

- Based on probability rules and “oriented randomness”
- Particularly effective in finding approximate global optimum for multimodal functions.
- Do not require gradient information.
- Function discontinuities have little effect.
- Resistant to becoming trapped in local optima.
- Can deal with large number of parameters.
- Provide a list of semi-optimum parameters instead of a single solution.

Stochastic Methods

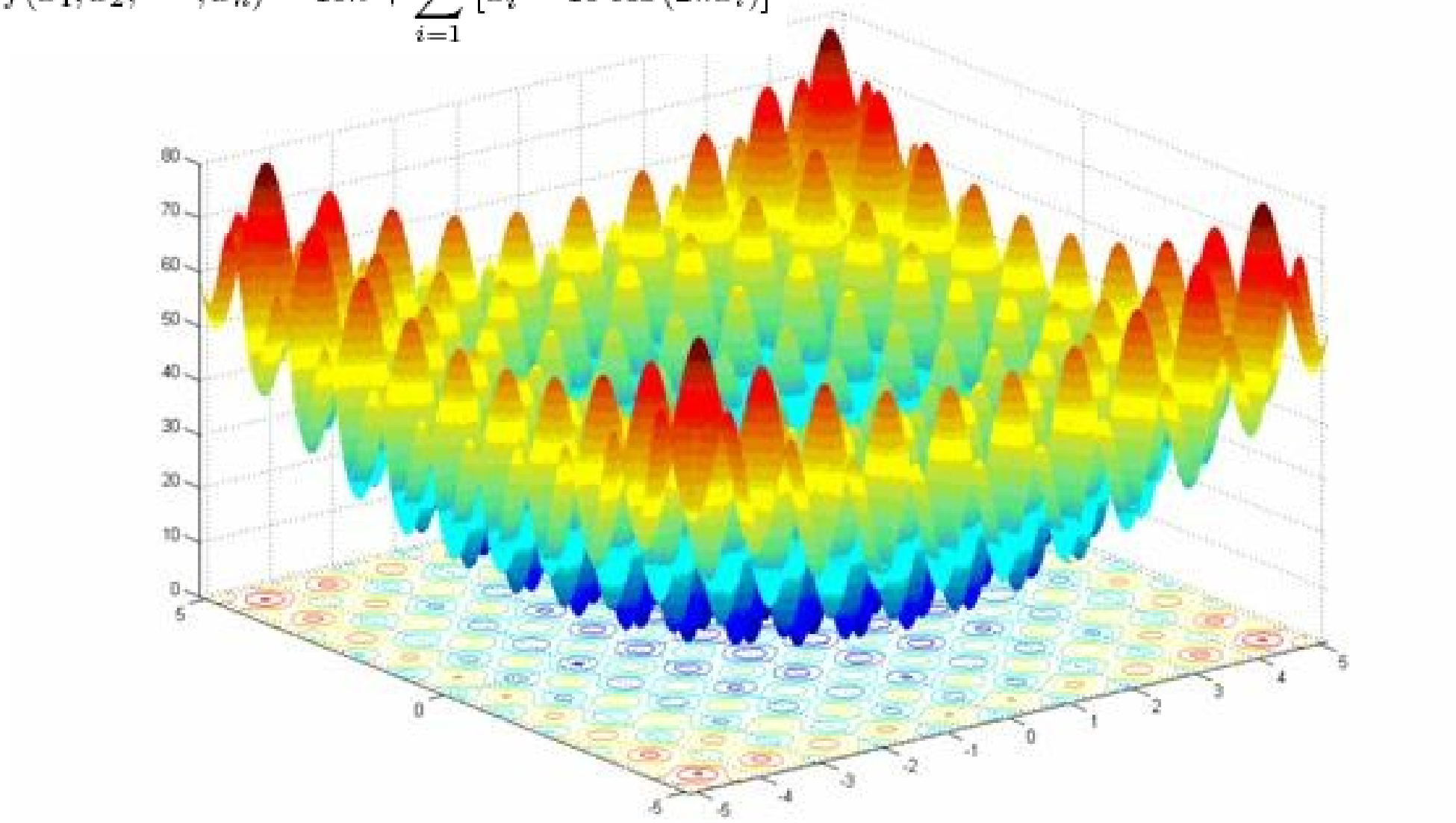
- Evolution Strategies (ESs)
- Genetic Algorithms (GAs)
- Simulated Annealing (SA)

Genetic Algorithms

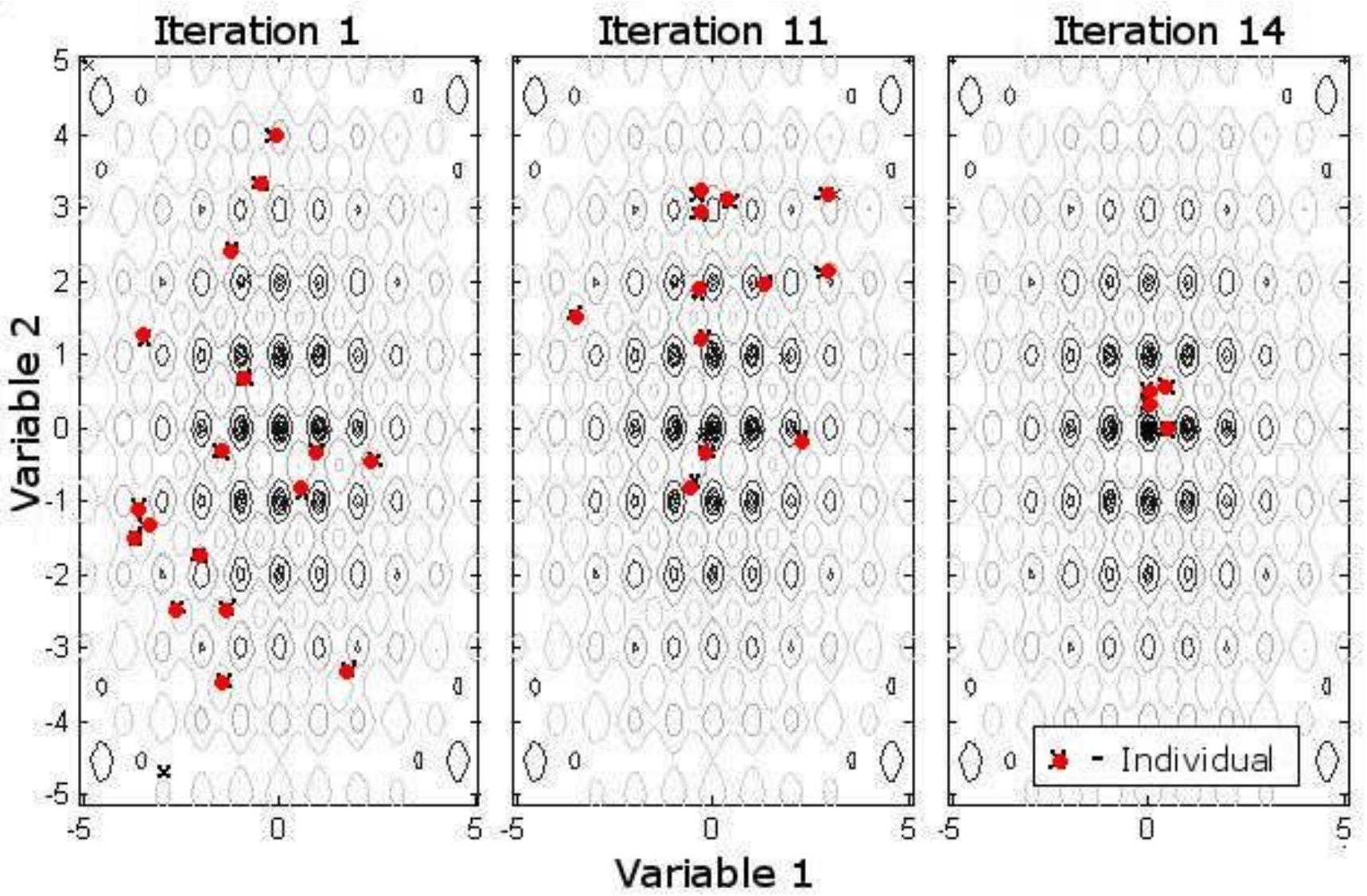
- Choose initial population
- Evaluate the fitness of each individual in the population (*fitness function*)
- Repeat
  - Select best-ranking individuals to reproduce
  - Breed new generation through crossover and mutation (genetic operations) and give birth to offspring
  - Evaluate the individual fitnesses of the offspring
  - Replace worst ranked part of population with offspring
  - Until termination

Rastringin function

$$f(x_1, x_2, \cdots, x_n) = 10n + \sum_{i=1}^{n} \left[x_i^2 - 10\cos(2\pi x_i) \right]$$



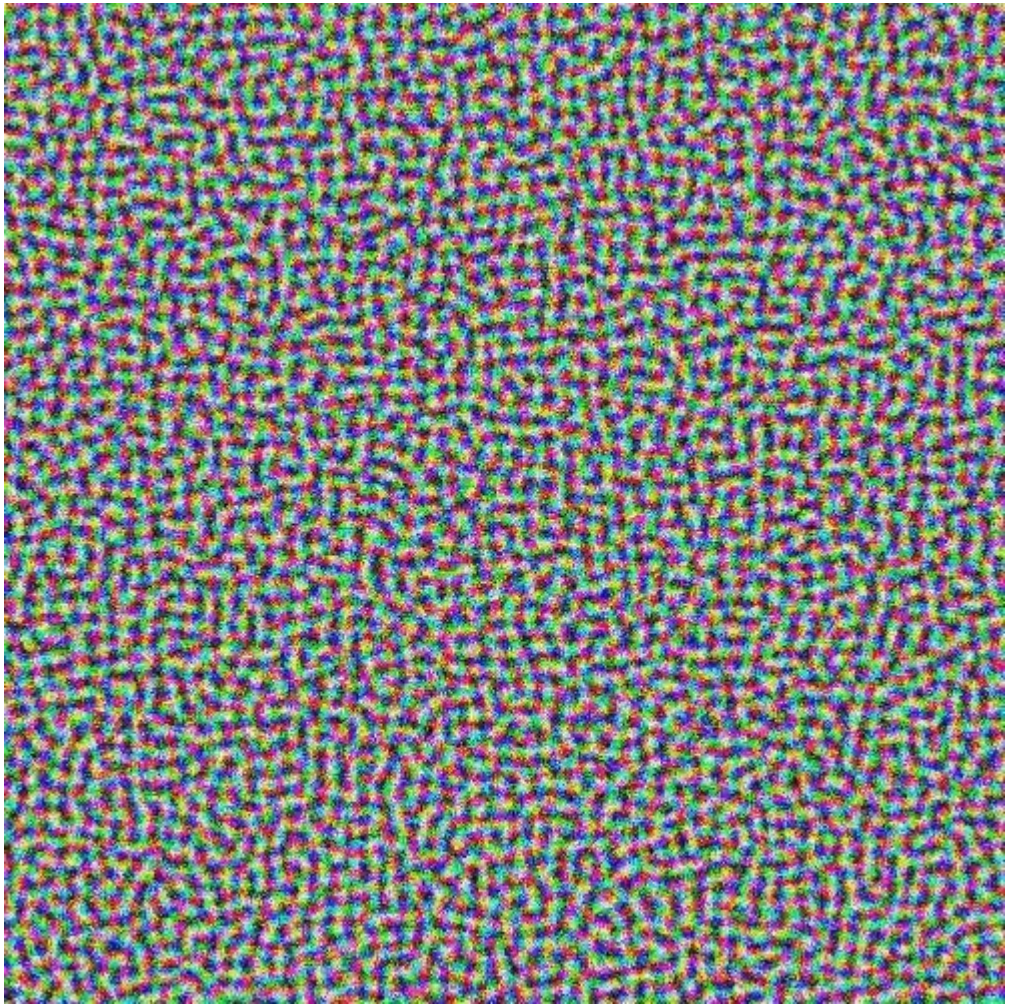
GAs in use


Iteration 1
Iteration 11
Iteration 14
Variable 2
Variable 1
- Individual

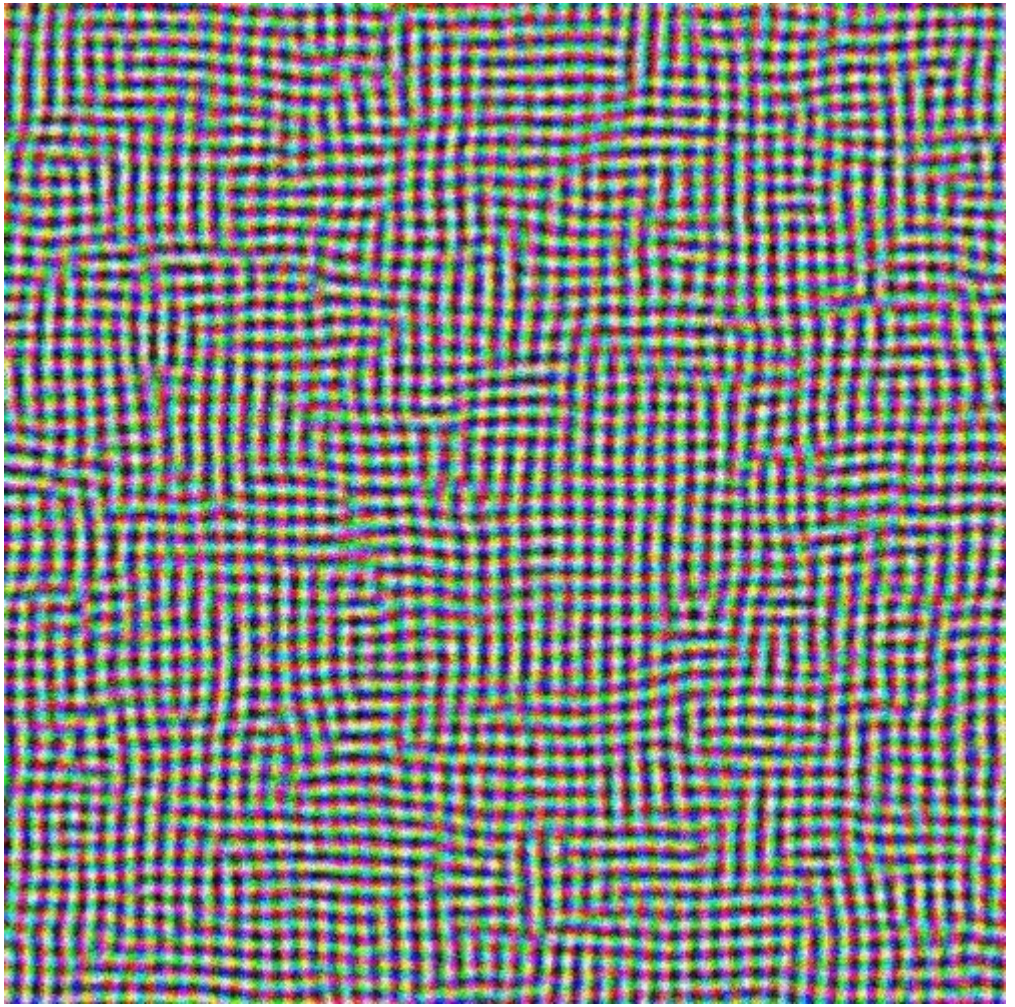
Simulated Annealing

- At the beginning, the system is in a state **s** with energy **E** and has a temperature **T**.
- System is taken into a new state **s'** and new energy **E'** is calculated. **If ΔE<0**, new state is accepted. If **ΔE>0** it is accepted with a probability given by the Boltzmann factor **exp -(ΔE/T)**.
- Temperature will be lowered and the whole process repeated until **T=0**.

Fast SA

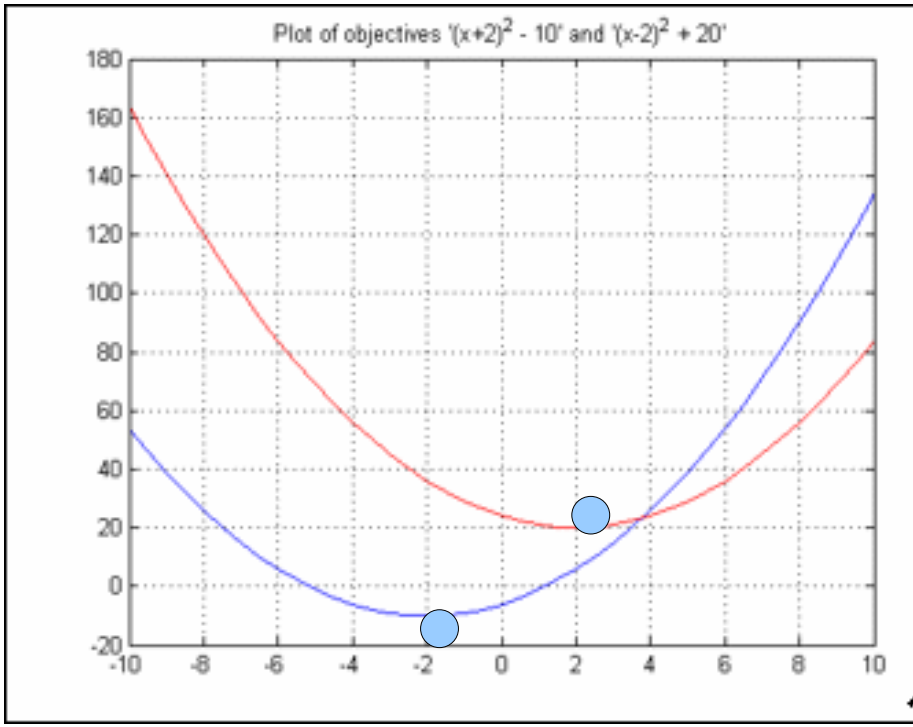


Slow SA

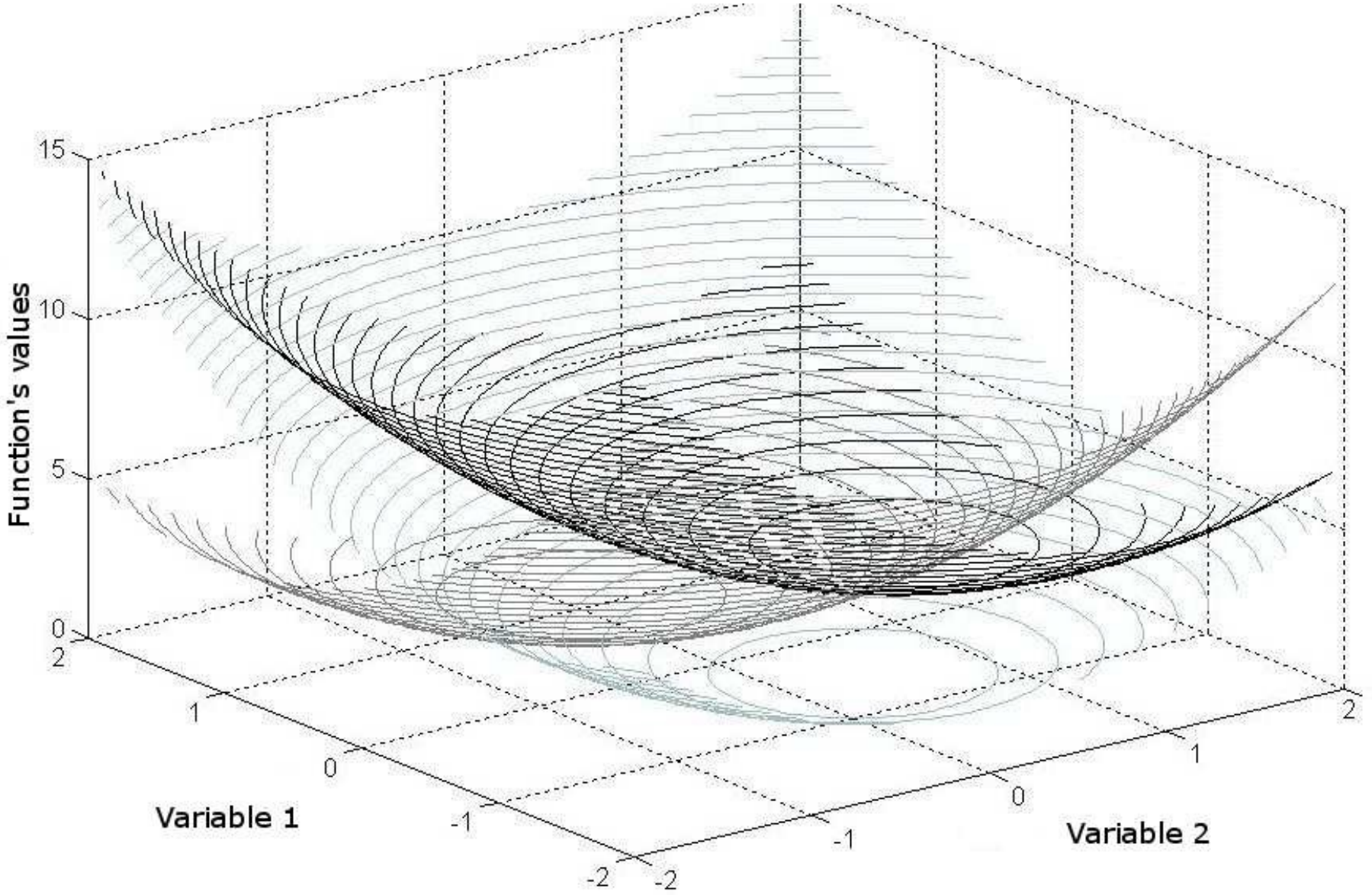


Multi-objective Approach

- The problem is formed by multiple objectives (functions).
- Two groups of solutions:
  - **Dominated Solutions** – solutions that are in all respects worse than others.
  - Non-dominated or **Pareto-optimal** solutions – solutions that are better than others in some aspects and worse in others.

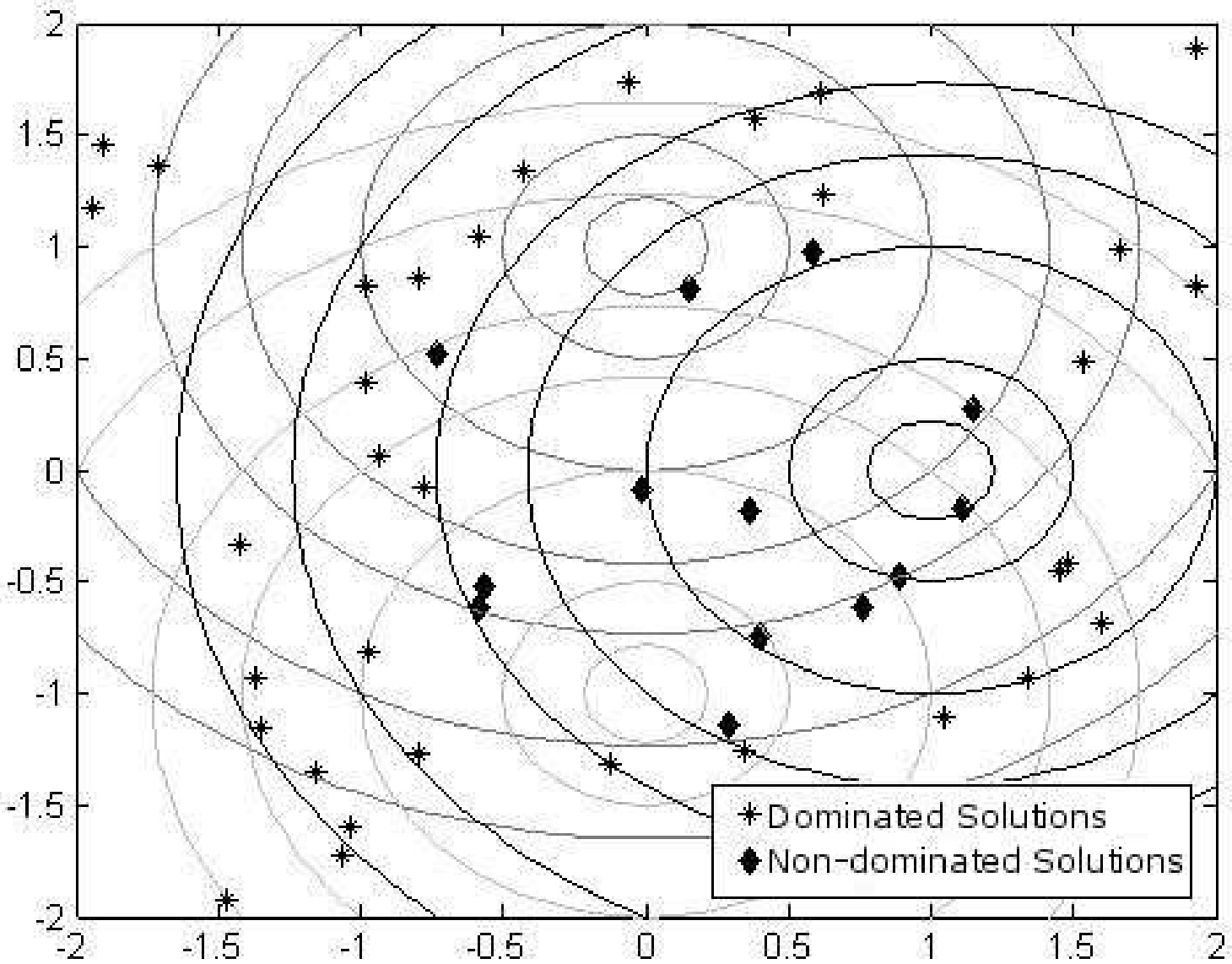


Multi-objective Approach

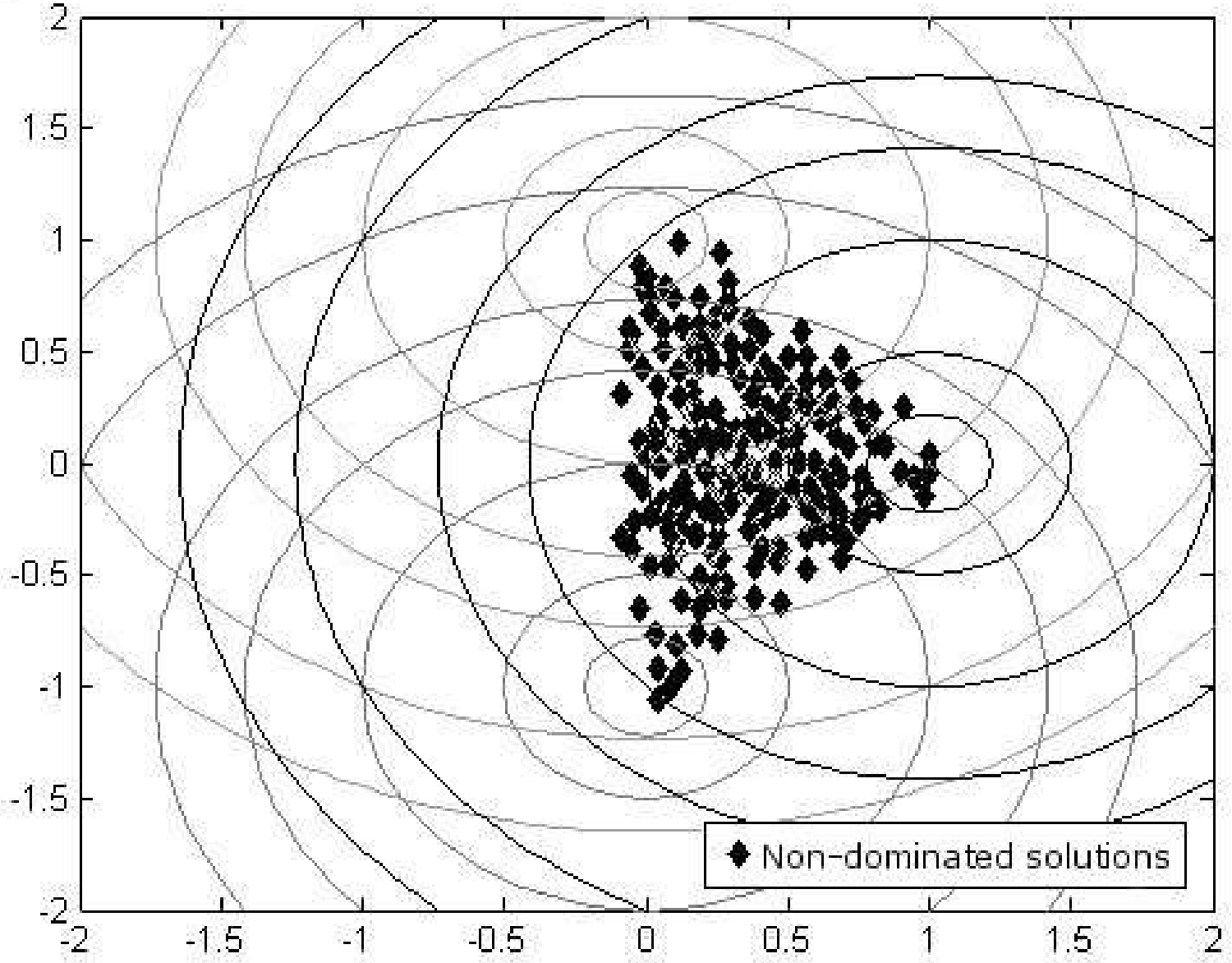


Minimization of three parabolas.

Beginning of the optimization process.



End of the optimization process.



References

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- http://en.wikipedia.org/wiki/Genetic_algorithm
- http://en.wikipedia.org/wiki/Evolution_strategy