Concurrent Systems Modeling using Petri Nets

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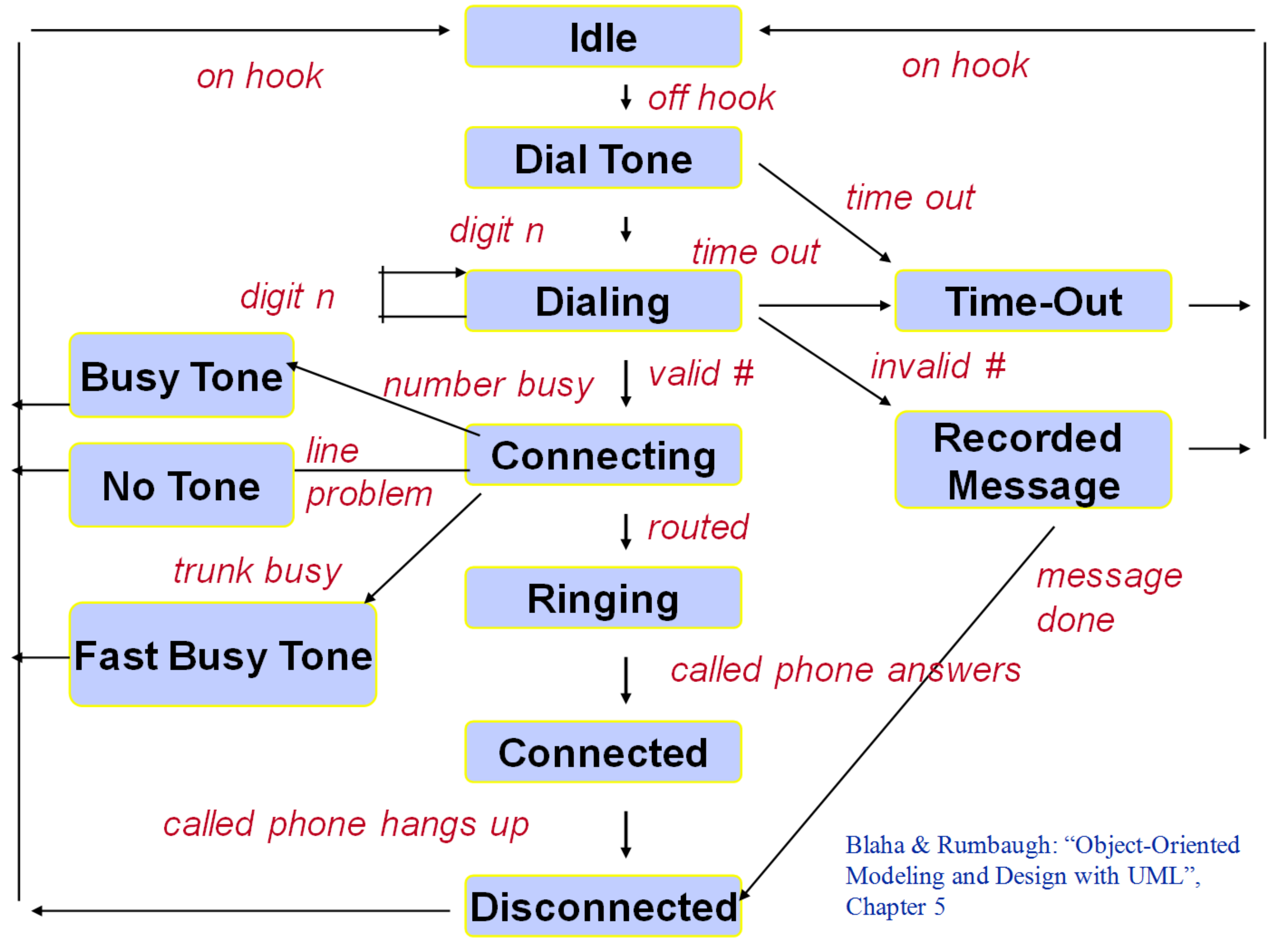
Based on lecture material by Marlon Dumas (University of Tartu, Estonia) and Wil van der Aalst

(Eindhoven University of Technology, The Netherlands http://www,workflowcourse.com)

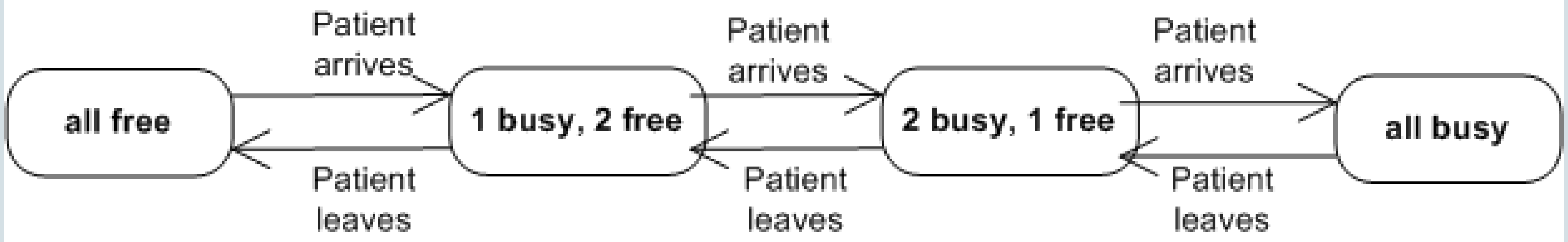
Behavior Modeling: State Machines



Blaha & Rumbaugh: "Object-Oriented Modeling and Design with UML", Chapter 5

Limitations of state machines

- Three doctors in a medical centre



- What if there are 6 doctors?
- What if a patient arrives and all doctors are busy?
- What if doctors can arrive and leave (so long as they are not busy)?
- State explosion…

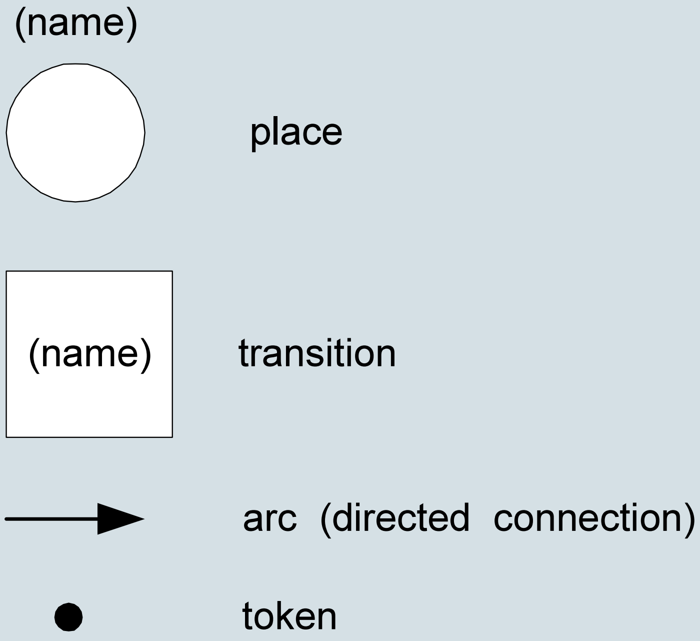
Concurrent systems modeling

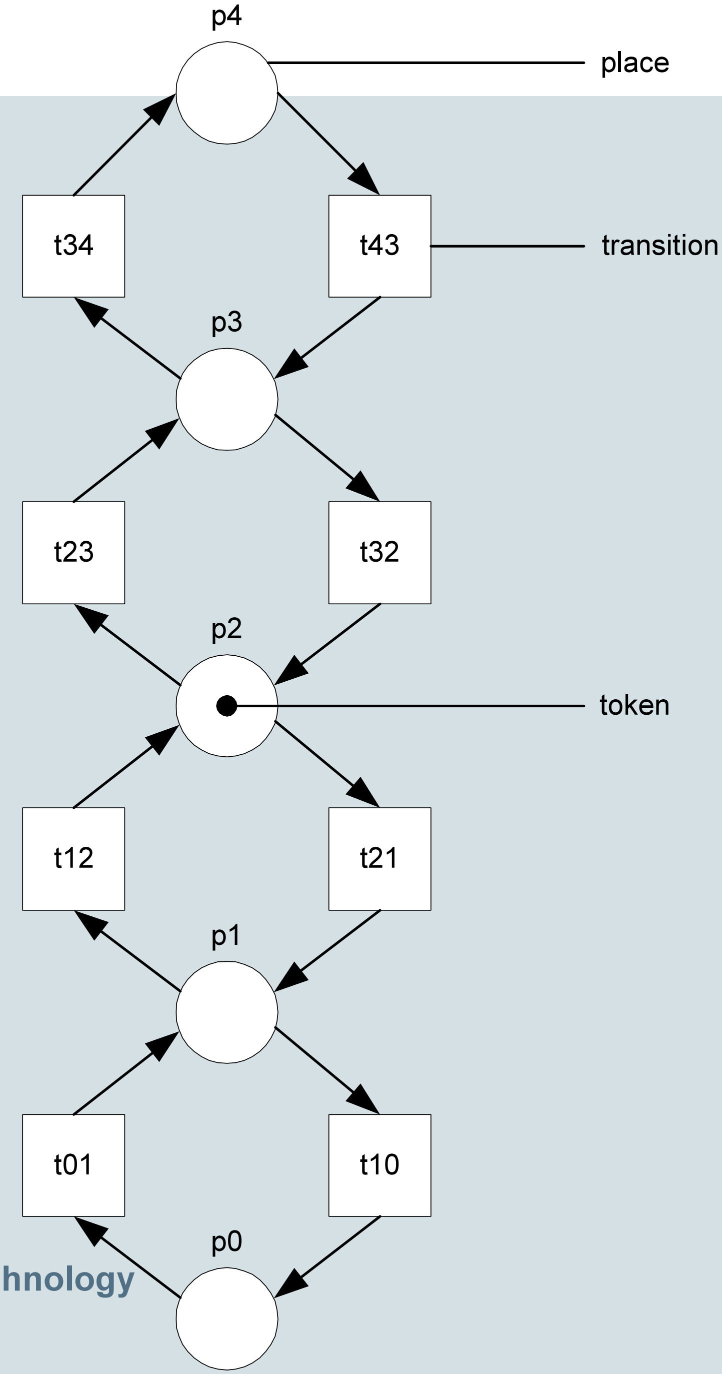
- State machines are useful to model behaviour of sequential systems
- But many systems are concurrent by nature
- Petri nets are a family of techniques for modeling systems with concurrency, communication and synchronization

Petri nets

- Simple technique for concurrent systems modeling
  - Four elements: **places**, **transitions**, **arcs** and **tokens**.
  - Graphical and mathematical description.
  - Formal semantics suitable for static analysis.
- Supported by verification and simulation tools (e.g. CPN Tools, ProM, LoLa, Woped).
- Once you understand Petri nets, you will be better equipped to understand other techniques for modeling systems with concurrency (e.g. process modeling notations)

Elements





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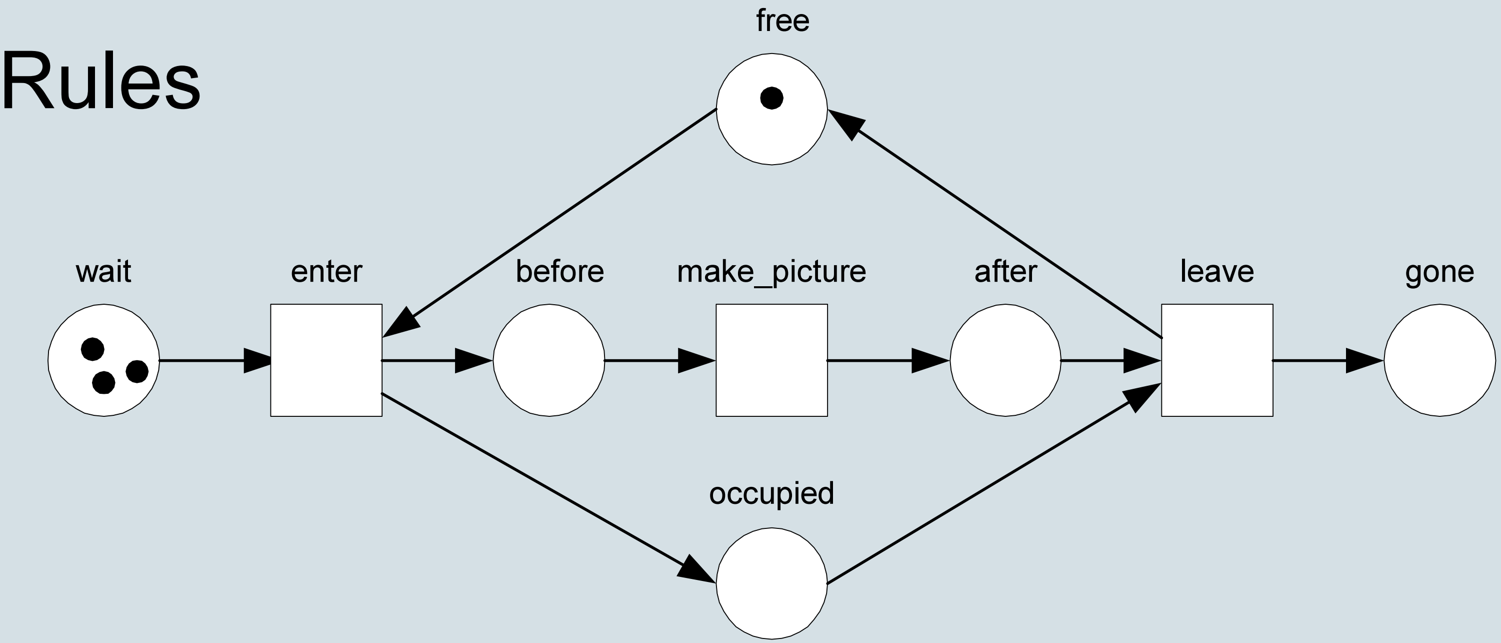
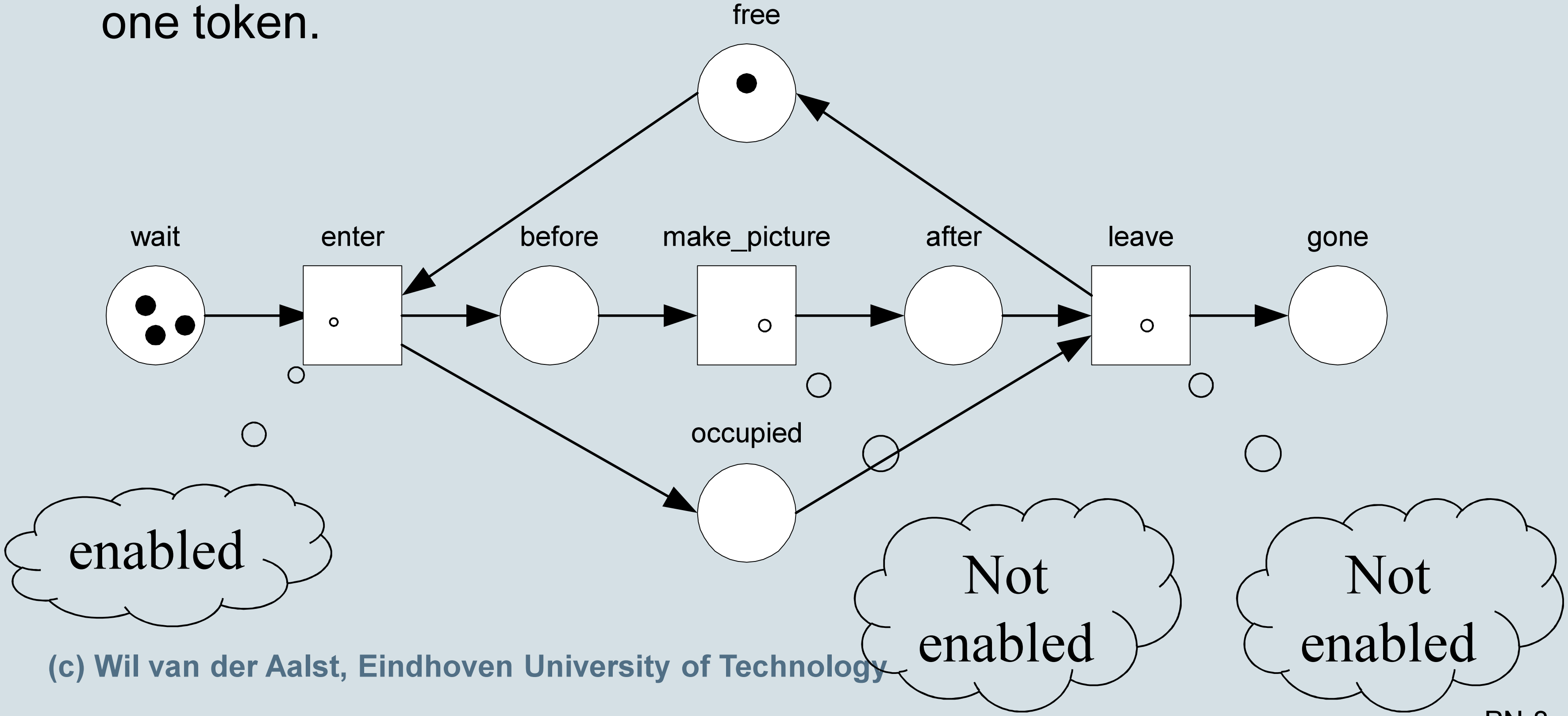
Rules



- Connections are directed.
- No connections between two places or two transitions.
- Places may hold zero or more tokens.
- First, we consider the case of at most one arc between two nodes.

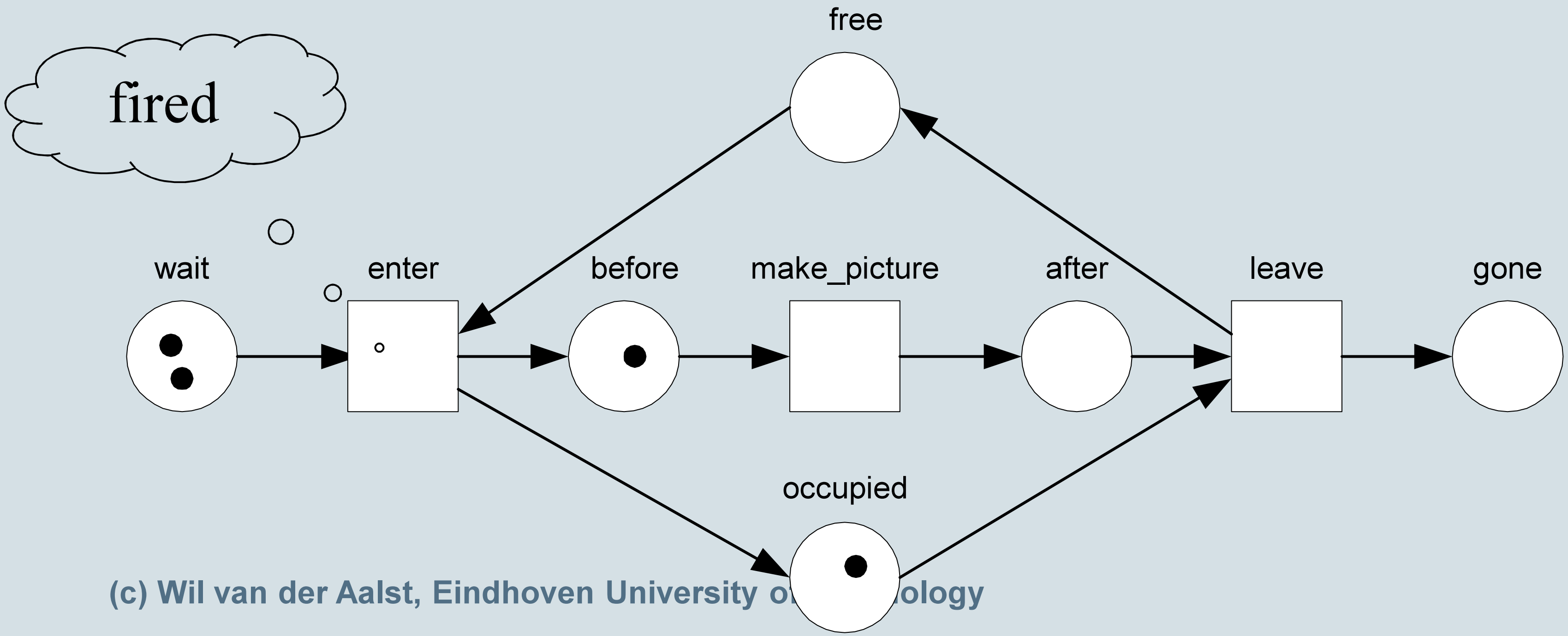
Marking and Enabled Transition

- The **state** of a net is a distribution of tokens over places (also referred to as **marking**).
- A transition is **enabled** if each of its input places contains at least one token.



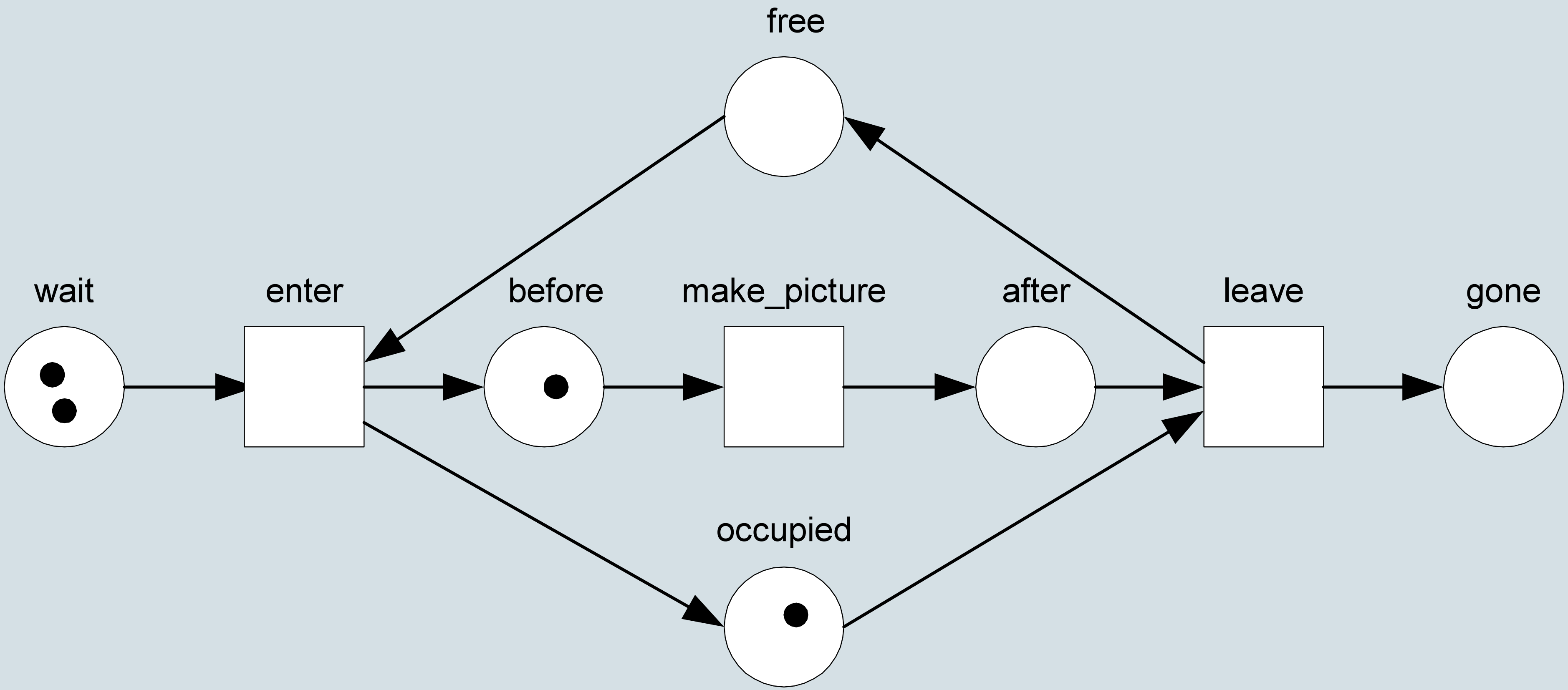
Firing

- An **enabled** transition can **fire** (i.e., it occurs).
- When it **fires** it **consumes** a token from each input place and **produces** a token for each output place.
- Which transitions are enabled now?



“Token Game”

- In the new state, *make_picture* is enabled. It will fire, etc.



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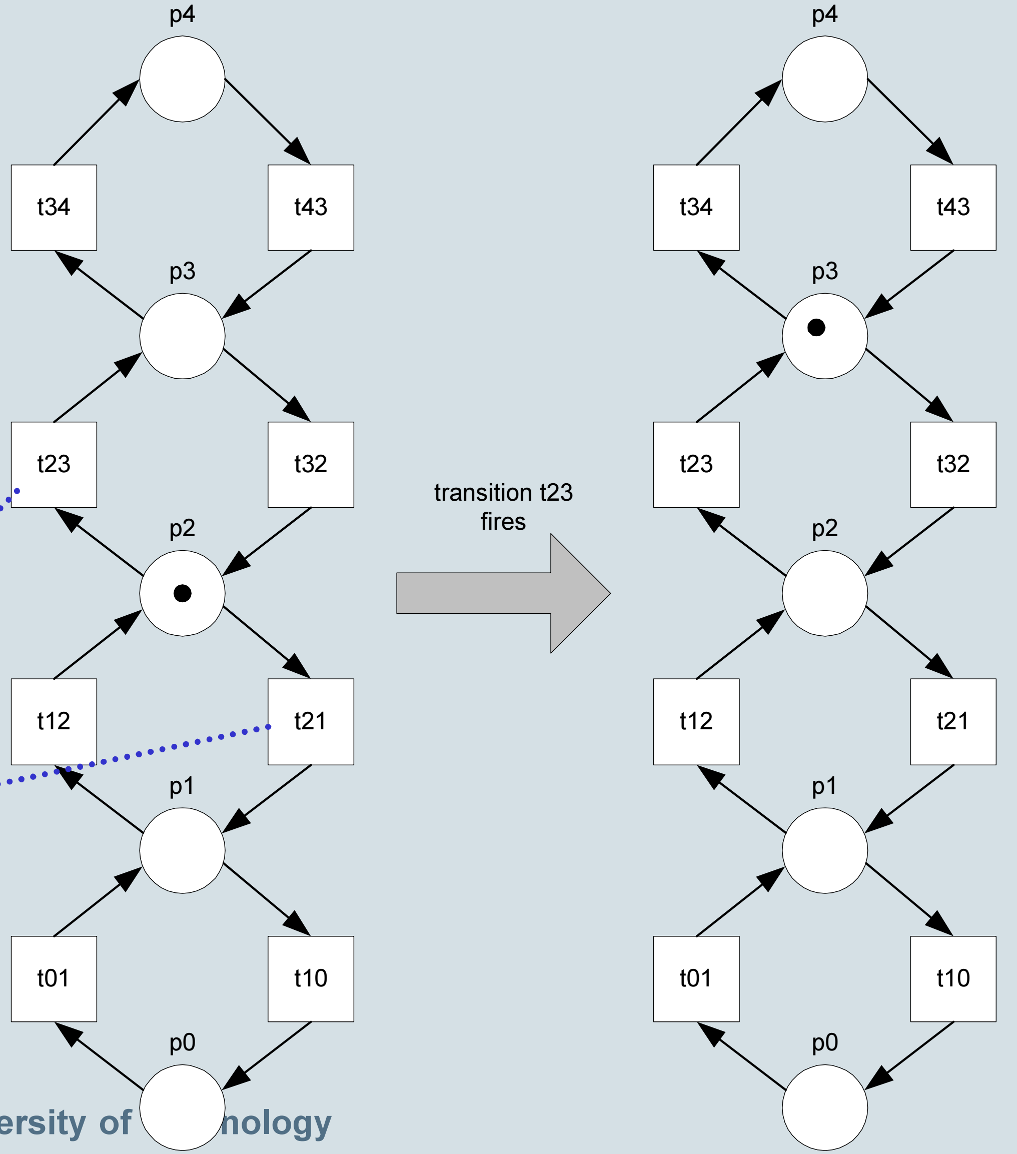
Remarks

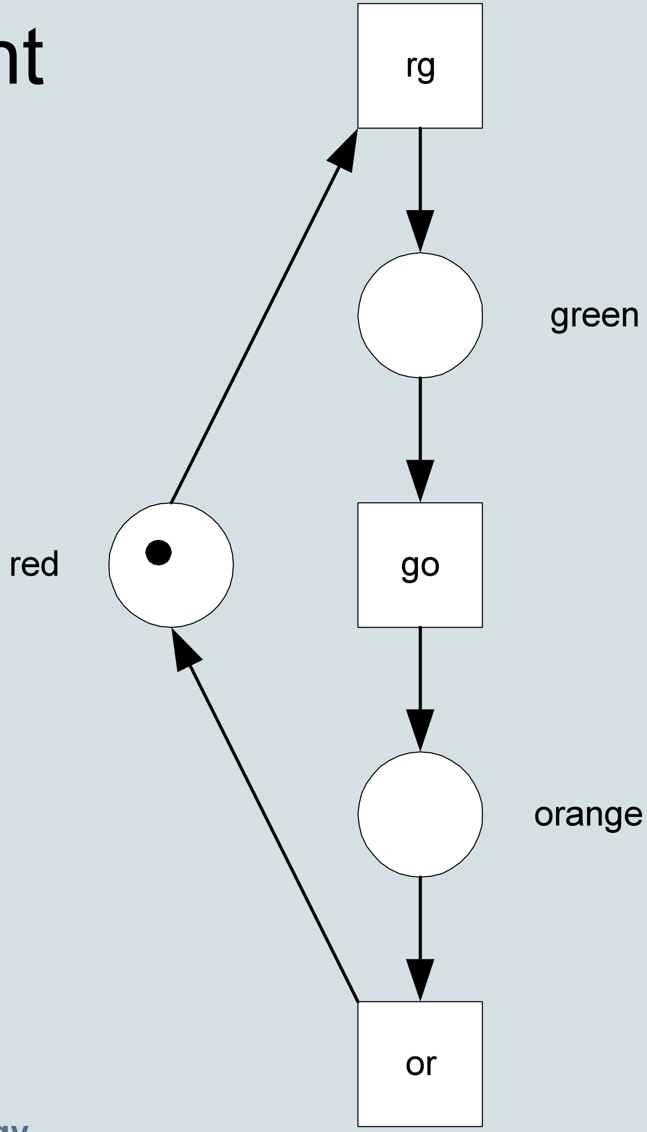
- Firing is **atomic**.
- Multiple transitions may be enabled, but only one fires at a time
- By default, choice is *non-deterministic*
- Any state machine can be trivially converted into a Petri net – How?

Non-determinism

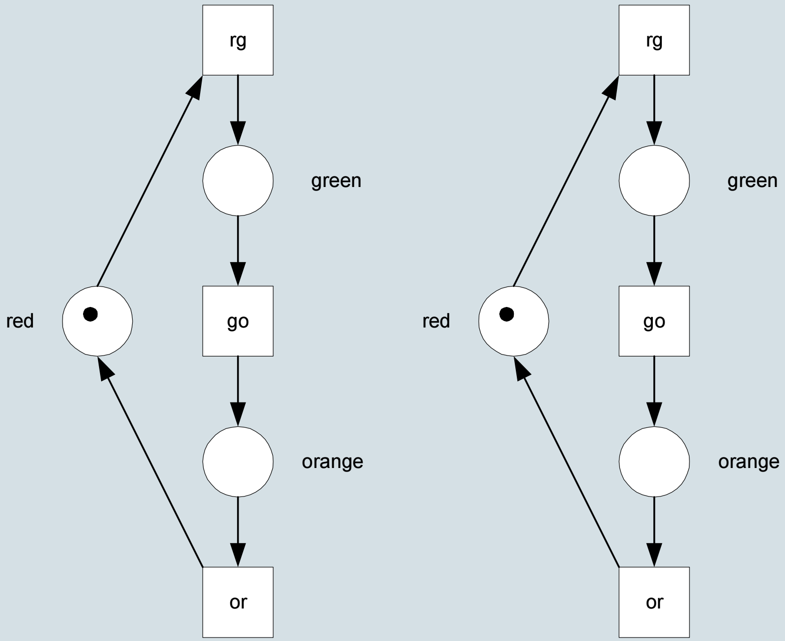


Two transitions are enabled but only one can fire

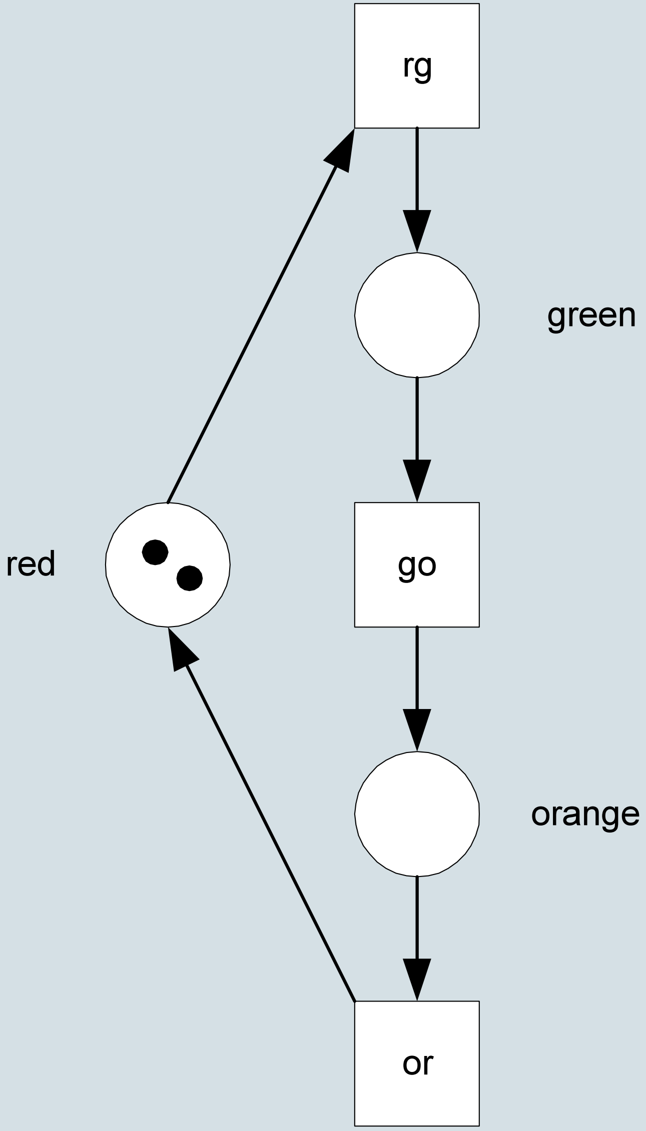
Example: Single traffic light



Two traffic lights



OR



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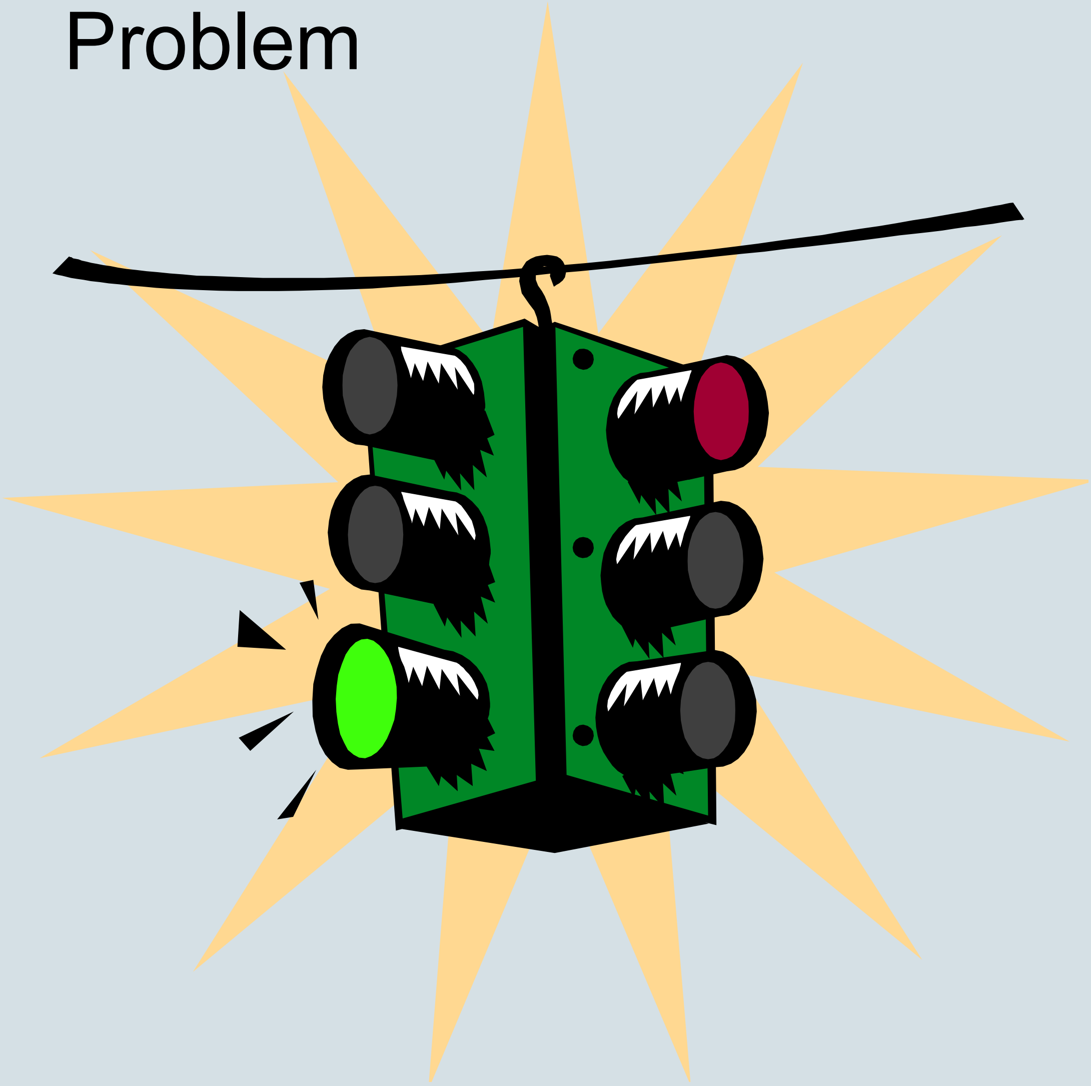
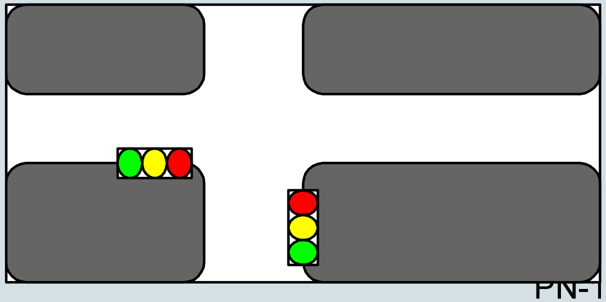
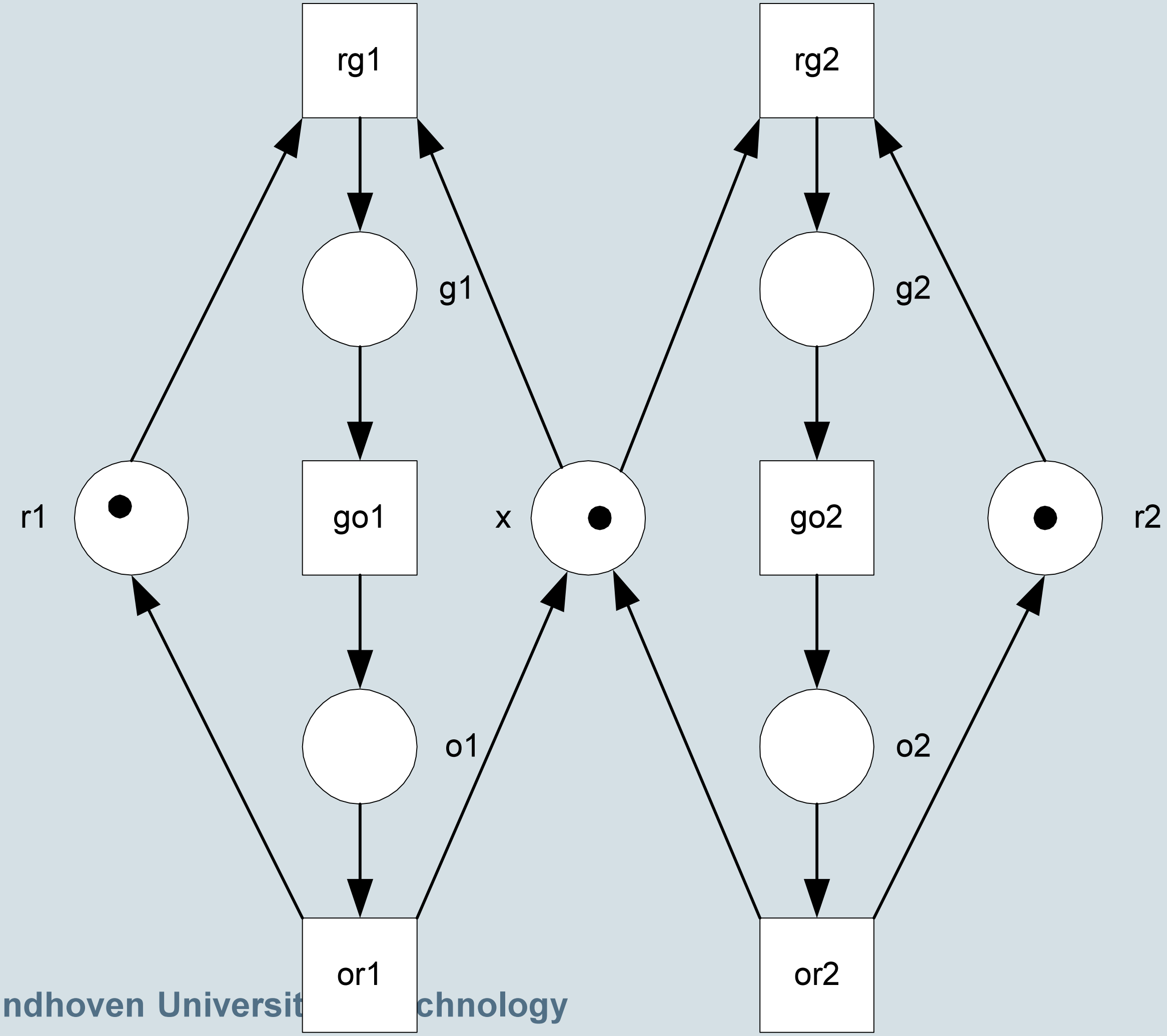
Problem





Solution

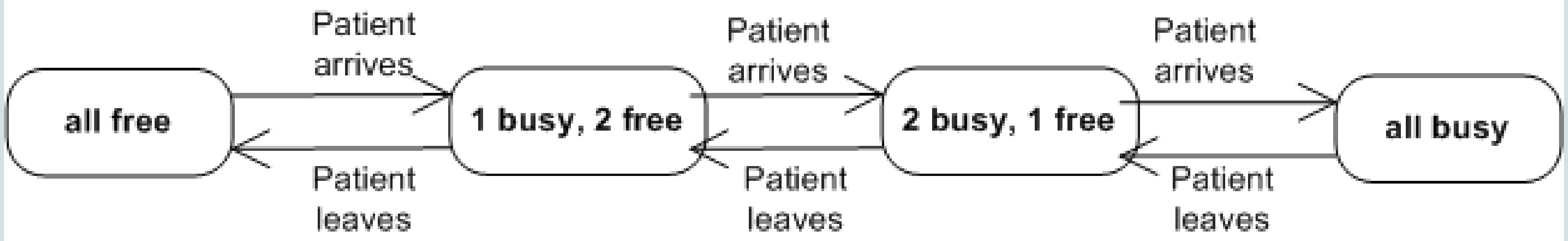


How to make them alternate?

Playing the "Token Game"

- FLASH animations:
  - http://wwwis.win.tue.nl/~wvdaalst/workflowcourse/
- Woped: A more sophisticated Petri net drawing and animation tool:
  http://www.woped.org/

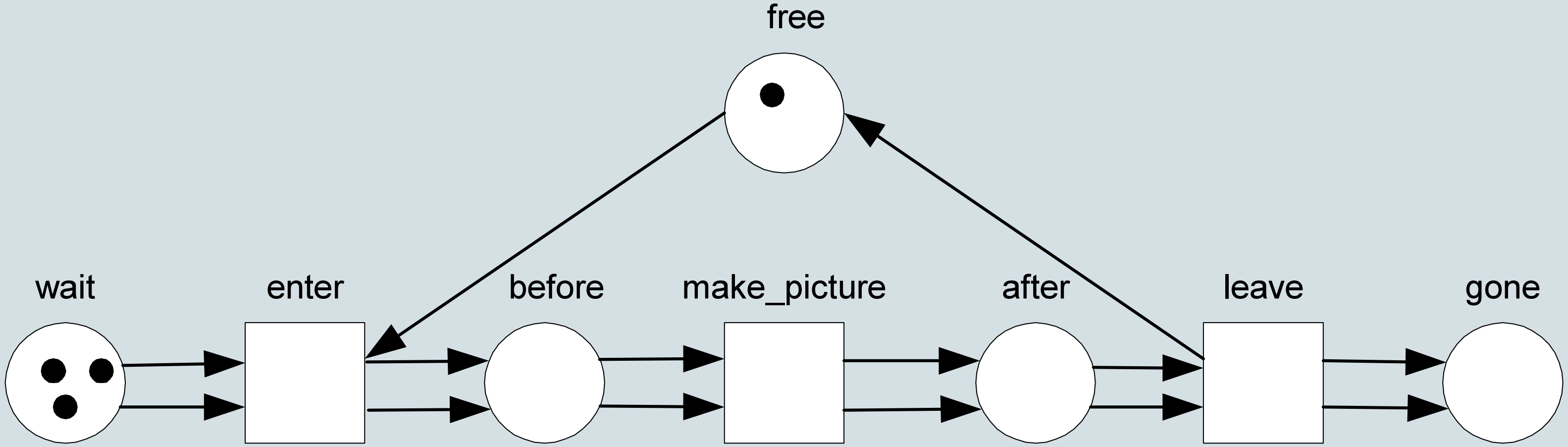
Exercise: Doctor’s scenario in Petri nets



Case 1: Patients arrive and leave, number of doctors fixed
Case 2: Patients arrive and leave, doctors arrive and leave (but only leave when they are free)
Case 3: When patients arrive, they are classified into simple and complex cases. Simple cases require only a doctor, complex cases require a doctor and a nurse. (Assume doctors and nurses do not arrive nor leave)

Multiple arcs connecting two nodes

- The number of arcs between an input place and a transition determines the number of tokens required to be enabled.
- The number of arcs determines the number of tokens to be consumed/produced.



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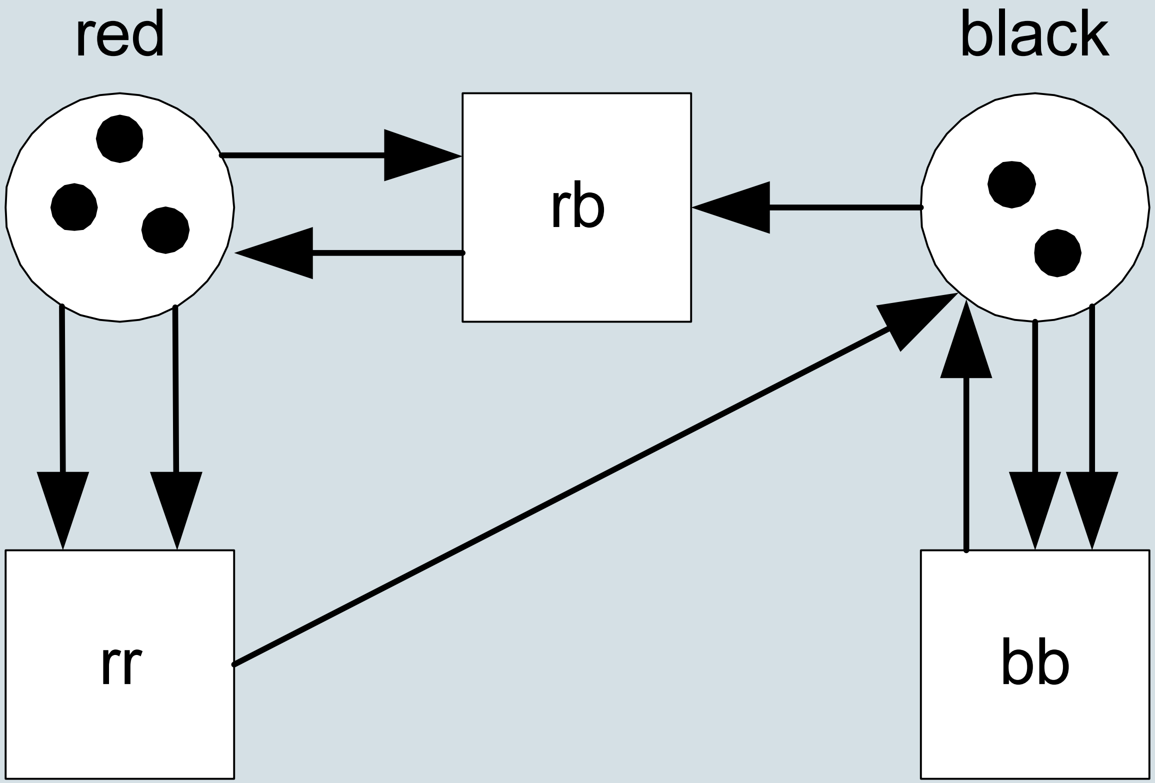
Example: Ball game



Which transition(s) is/are enabled?

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You should be able to ...

- Explain what is a Petri net and what are the basic elements of (plain) Petri nets
- Play a token game on a Petri net.
- Model simple concurrent systems using Petri nets.