The Targeted Depletion Benchmark (TDB): A Stochastic Optimization Model for Labor Cessation and Sequence Risk

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The Problem: SRR and Over-Accumulation

Traditional lifecycle models and modern FIRE heuristics mandate continuous capital accumulation until decumulation to mitigate Sequence of Returns Risk (SRR).

However, this creates a systemic inefficiency: **agents over-accumulate capital**, exacting a severe tax on utility-adjusted labor costs.

The Shift in Objective: Mid-stage accumulation is widely assumed to be a return-maximization problem. We demonstrate it is actually a **risk-minimization problem**. Unmitigated portfolio variance (σ) exponentially increases the upfront capital required to safely cease labor prior to retirement.

The Stochastic TDB Framework

The Stochastic Targeted Depletion Benchmark (W^*_{TDB}) formalizes the "Coasting" phase (savings cessation) under log-normal market constraints.

1. The Wealth Accumulation Process: Assuming continuously compounded returns map to a geometric drift parameter (μ_g), wealth accumulation over duration T_{coast} is:

$$W_T = W_0 \exp\left(\sum_{t=1}^{T_{coast}} r_t\right) \quad \text{where} \quad r_t \sim N\left(\ln(1+\mu) - \frac{\sigma^2}{2}, \sigma^2\right)$$

2. The Optimization Objective: We resolve the minimum initial capital (W_0) required to reach terminal retirement liability (W_{ret}) subject to a prescribed confidence interval ($1-\alpha$):

$$W^*_{TDB} = \min W_0 \quad \text{s.t.} \quad P(W_T \geq W_{ret}) \geq 1-\alpha$$

Stochastic Optimization Surface

Using **Wolfram Mathematica**, we modeled the non-linear penalty of volatility drag and accumulation duration to resolve the constraint boundaries.

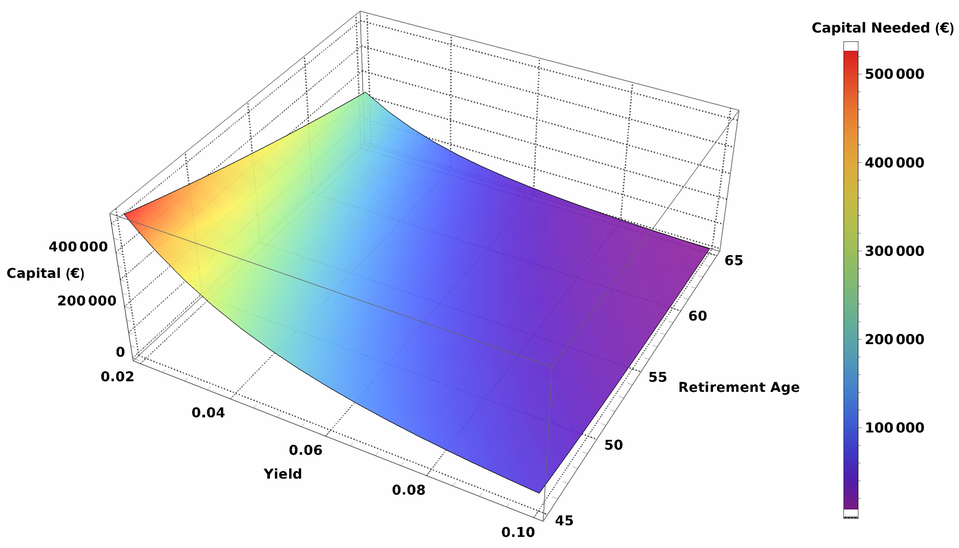

Fig 1. TDB Solution Surface: Required wealth scaling exponentially. Computed and rendered via Wolfram Mathematica.

• **Geometric Decay:** The upward curvature visually proves that relying on volatile drift heavily limits geometric decay, defining an explicit efficiency curve for bounded lifetime labor deployment.

The Efficient Frontier of Accumulation

Counter-intuitively to traditional long-horizon advice (which favors 100% equity), geometric variance severely penalizes early labor cessation. Optimization reveals a U-shaped **Efficient Frontier of Accumulation**.

Portfolio Benchmark	Expected μ	Volatility (σ)	Required W^*_{TDB}
100% Equity Strategy	7.0%	15.0%	$324,827
80/20 Aggressive	**6.5%**	**12.0%**	**$268,230**
60/40 Moderate	5.5%	9.0%	$271,250

Targeting a 90% confidence threshold. Variance reduction systematically commands priority over absolute return targets to minimize required labor.

Empirical Backtest (1872 - 1989)

To test the theoretical geometric framework against clustered historical tail risks, the dynamic unified model was backtested across **118 overlapping 35-year cohorts** using Robert Shiller's historical dataset (real equity and bond returns).

Metric	Empirical Outcome
Dynamic Historical Success Rate	**91.5%** (108/118 Cohorts)
95% Wilson Score Interval	[85.1%, 95.3%]
Blanchett Adjusted Target (W_{ret})	$648,678
Max Emergency Labor (Worst Cohort)	0 years

Robustness: Despite empirical trade-offs, buffering from starting yields allowed severe cohorts (e.g., the 1929 Great Depression) to conclude safely, preventing the realization of catastrophic margin calls across all 118 historical trials.

Interactive TDB Simulation

Scan to run the open-source Streamlit app:

tdb-calculator.streamlit.app

Published SSRN Working Paper

Scan to read the full empirical study

ssrn.com/abstract=6556206

GitHub: github.com/Paul-HenryP/Targeted-Depletion-Benchmark