SENSITIVITY ANALYSIS
TWO TYPES OF SENSITIVITY

Value sensitivity - variations in the maximum expected utility

Decision sensitivity - changes in optimal strategy
THE OIL WILDCATTER PROBLEM

- Hole is good - $260,000
- P(Hole = good) = 0.2
- Drilling - $60,000
- Test - $5,000
- False positive rate - 0.05
TO DRILL OR NOT TO DRILL?

- Hole is good - $260,000
- $P(Hole = \text{good}) = 0.2$
- Drilling - $60,000$
- Test - $5,000$
- False positive rate - 0.05
- Hole is good - $260,000
- $P(\text{hole is good}) = 0.2$
- Drilling - $60,000$
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**Optimal strategy** - test and then drill iff test is positive
WHAT IF HE IS UNCERTAIN?

- Hole is good - $260,000?
- \( P(\text{hole is good}) = 0.2? \)
- Drilling - $60,000
- Test - $5,000
- False positive rate - 0.05
POLICIES FOR THE OPTIMAL STRATEGY

- $\delta_{\text{Test?}} = y$ for $\text{Test?}$
- $\delta_{\text{Drill?}}(\text{Test?}, T')$
  - $\delta_{\text{Drill?}}(y, \text{pos}) = y$
  - $\delta_{\text{Drill?}}(y, \text{neg}) = n$
  - $\delta_{\text{Drill?}}(n, \text{no-test}) = n$

$t = P(\text{Hole} = \text{good})$

$s = \text{Gain}(\text{Hole} = \text{good}) - 60000$

$\delta_{\text{Drill?}}$ is optimal for $(t, s) = (0.2, 200000)$
WHICH PARAMETER VALUES SUPPORT THIS POLICY?

Optimal strategy - test and then drill iff test is positive

For which $t$ and $s$ it still holds?

$$
\begin{align*}
\text{EU}(\text{Drill} \mid n, \text{no-test}) & = (P(\text{good} \mid \text{no-test})s - P(\text{bad} \mid \text{no-test})60000, 0) \\
& = (ts - (1 - t)60000, 0), \\
\text{EU}(\text{Drill} \mid y, \text{pos}) & = (P(\text{good} \mid \text{pos})s - P(\text{bad} \mid \text{pos})60000, 0) \\
& = \left(\frac{ts - 0.05(1 - t)60000}{0.95t + 0.05}, 0\right), \\
\text{EU}(\text{Drill} \mid y, \text{neg}) & = (P(\text{good} \mid \text{neg})s - P(\text{bad} \mid \text{neg})60000, 0) \\
& = (-60000, 0).
\end{align*}
$$
The policy $\delta_{Drill?}$ is optimal if

$$\text{EU}(\text{Drill?} = n \mid n, \text{no-test}) \geq \text{EU}(\text{Drill?} = y \mid n, \text{no-test}),$$
$$\text{EU}(\text{Drill?} = y \mid y, \text{pos}) \geq \text{EU}(\text{Drill?} = n \mid y, \text{pos}),$$
$$\text{EU}(\text{Drill?} = n \mid y, \text{neg}) \geq \text{EU}(\text{Drill?} = y \mid y, \text{neg}).$$

This gives the following inequalities:

$$0 \geq ts - (1 - t)60000,$$
$$0 \leq ts - 0.05(1 - t)60000,$$
$$0 \geq -6000.$$

That is,

$$ts + 3000t - 3000 \geq 0 \geq ts + 60000t - 60000.$$
ANALYZING THE FIRST DECISION

Substitute *Drill?* with its chance-node representation
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\[
\text{EU}(\text{Test}\,? = y) = -5000 + P(\text{pos})(\text{EU}(\text{Drill}\,? = y \,|\, \text{pos}) ) \\
= -5000 + (0.95t + 0.05) \frac{ts - 0.05(1 - t)60000}{0.95t + 0.05} \\
= ts + 3000t - 8000, \\
\text{EU}(\text{Test}\,? = n) = 0.
\]

This yields that testing is optimal if

\[ts + 3000t - 8000 \geq 0.\]

Admissible domains:

For \( s = 200000 \): \( t \geq \frac{8}{203} \)

for \( t = 0.2 \): \( s \geq 37000 \)
Admissible domains:
For $s = 200000$ : $\frac{3}{203} \leq t \leq \frac{3}{13}$
for $t = 0.2$ : $12000 \leq s \leq 240000$

$\text{Admissible domains:}$
$\text{For } s = 200000 : \quad t \geq \frac{8}{203}$
for $t = 0.2 : \quad s \geq 37000$

$t \in [\frac{8}{203}, \frac{3}{13}]$
$s \in [37000, 240000]$
Theorem 11.3. Let $s$ be a utility parameter in the influence diagram ID, let $D$ be the last decision in ID, and let $\pi$ be any configuration of the required past of $D$. Then for any $d$ in $D$, the expected utility of $d$ given $\pi$ is a linear function in $s$.

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Solving the influence diagram with $s = 200000$ we get the following expected utilities:

$$\text{EU}(\text{Drill?} \mid \text{pos}) = (156666, 0),$$
$$\text{EU}(\text{Drill?} \mid \text{neg}) = (-60000, 0),$$
$$\text{EU}(\text{Drill?} \mid \text{no-test}) = (-8000, 0),$$
$$\text{EU}(\text{Test?}) = (32600, 0).$$

Changing $s$ to $150000$ we get

$$\text{EU}(\text{Drill?} \mid \text{pos}) = (115000, 0),$$
$$\text{EU}(\text{Drill?} \mid \text{neg}) = (-60000, 0),$$
$$\text{EU}(\text{Drill?} \mid \text{no-test}) = (-18000, 0),$$
$$\text{EU}(\text{Test?}) = (22600, 0).$$
ONLY 2 VALUES ARE NEEDED

This yields the following expressions:

\[
\begin{align*}
\text{EU}(\text{Drill?} = y \mid \text{pos}) &= 0.833s + 10000, \\
\text{EU}(\text{Drill?} = y \mid \text{neg}) &= -60000, \\
\text{EU}(\text{Drill?} = y \mid \text{no-test}) &= 0.2s - 48000, \\
\text{EU}(\text{Test?} = y) &= 0.2s - 7400,
\end{align*}
\]

which are the same as the result of the expressions in Section 11.3.1.
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\text{EU}(\text{Drill}? = y \mid \text{pos}) &= 0.833s + 10000, \\
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\text{EU}(\text{Drill}? \mid n, \text{no-test}) &= (P(\text{good} \mid \text{no-test})s - P(\text{bad} \mid \text{no-test})60000, 0) \\
&= (ts - (1 - t)60000, 0), \\
\text{EU}(\text{Drill}? \mid y, \text{pos}) &= (P(\text{good} \mid \text{pos})s - P(\text{bad} \mid \text{pos})60000, 0) \\
&= \left( \frac{ts - 0.05(1 - t)60000}{0.95t + 0.05}, 0 \right), \\
\text{EU}(\text{Drill}? \mid y, \text{neg}) &= (P(\text{good} \mid \text{neg})s - P(\text{bad} \mid \text{neg})60000, 0) \\
&= (-60000, 0).
\end{align*}
\]
For a probability parameter $t$ and a utility parameter $s$, the expected utilities have the form $as+b$, where $a$ and $b$ are fractions of linear expressions over $t$. This means that there are 8 coefficients to determine.