Quantum Cryptography (fall 2013) Prof. Dr. Dominique Unruh

Exercise Sheet 8

Out: October 31, 2013 Due: November 5, 2013

• Note: This homework will not give the usual 20 points, but only 8 points. •

Problem 1: Schmidt Decomposition

(a) For a given state $|\Psi\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B$, the Schmidt number is the smallest n such that a Schmidt decomposition $|\Psi\rangle = \sum_{i=1}^{n} \lambda_i |\alpha_i\rangle|\beta_i\rangle$ exists.

We call a state $|\Psi\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B$ entangled if $|\Psi\rangle$ *cannot* be written as $|\Psi\rangle = |\Psi_A\rangle \otimes |\Psi_B\rangle$.

Show that a state is entangled if and only if it has Schmidt number greater than 1. (This justifies using the Schmidt number as a measure of how entangled a state is.)

(b) Let a state $|\Psi\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B$ be given. Assume for simplicity that $\dim \mathcal{H}_A = \dim \mathcal{H}_B$. Show that $\mathrm{tr}_A |\Psi\rangle\langle\Psi|$ and $\mathrm{tr}_B |\Psi\rangle\langle\Psi|$ have the same eigenvalues.

Hint: Represent $|\Psi\rangle$ in its Schmidt decomposition. Then compute the partial trace tr_A and tr_B directly on that representation.