You will need 50% of all homework points to qualify for the exam. (That is, if you get at least 50%, your final grade will be the exam grade. And if you do not get 50%, you do not pass the course.)

You may hand in your solutions in person or by email. If you submit by email, either scan a handwritten solution or typeset your solution readably. I do not consider ASCII formulas readable.

When submitting, indicate your name and your matriculation number. On your first submission, please also indicate a password, this password will be needed for accessing the solutions and your points online.

Problem 1: Addition on a Turing machine

Construct a Turing machine that does the following: Given input $a|b$ (where $a$ and $b$ are unsigned integers in binary encoding and $|$ is a special symbol), it outputs the sum of $a$ and $b$ (in binary encoding).

Explicitly give the set $\Gamma$ of symbols (containing at least $\triangleright$, $\square$, $0$, $1$, $|$), the set of states $Q$, and the transition function $\delta$.

You do not need to prove that your TM does add correctly, but please add sufficient comments to make the workings understandable.

Notes: Input $a|b$ means that the input tape contains $\triangleright a|b\square\square\ldots$. You can choose whether your integers are encoded lsb-first or msb-first, but make your choice explicit and stick to it.

Problem 2: NP-problems

Out of the following five problems, three are in $\text{NP}$. The other two are not (or at least, science cannot currently show that they are).

- Identify those three.
- Show that they are in $\text{NP}$. That is, say what the Turing machine $M$ from the definition of the class $\text{NP}$ does, say what $u$ is. You do not need to “program” $M$, it is sufficient to say what $M$ does.\footnote{E.g., like “$M(x, w)$ outputs 1 iff $|x| = |w|$.”}
- For the remaining two, explain why you cannot show that they are in $\text{NP}$. (No formal proof is needed. Just an explanation of the difficulties.)
(a) \textbf{SAT} := \{B : B is a satisfiable Boolean formula\}. A Boolean formula is a formula with variables \(x_1, x_2, x_3, \ldots\) and the operations \(\land, \lor, \neg\). A Boolean formula is satisfiable if the variables \(x_1, x_2, \ldots\) can be assigned values true/false so that the formula evaluates to true.

(b) \textbf{PALIN} := \{x : x is a palindrome\}.

(c) \textbf{EQ} \_1 := \{(p_1, \ldots, p_n) : \exists x_1, \ldots, x_m \in \mathbb{Z}. p_1(x_1, \ldots, x_m) = \cdots = p_n(x_1, \ldots, x_m) = 0\}.

Here \(p_1, \ldots, p_n\) are polynomials in \(m\) variables with integer coefficients. (Both \(n, m\) can vary.)

(d) \textbf{EQ} \_2 := \{(p_1, \ldots, p_n, b_1, \ldots, b_n) : \exists x_1, \ldots, x_m \in \mathbb{Z} \text{ s.t. } p_1(x_1, \ldots, x_m) = \cdots = p_n(x_1, \ldots, x_m) = 0 \land |x_1| \leq b_1, \ldots, |x_m| \leq b_m\}.

Here \(p_1, \ldots, p_n\) are polynomials in \(m\) variables with integer coefficients. (Both \(n, m\) can vary.)

(e) \textbf{co-SAT} := \{B : B is not a satisfiable Boolean formula\}. 