Lab 1 - due 20 September 2017

Your Name

September 6, 2017
References in your solutions

Please add references to any external material that you base your answers on.

Typesetting

Please put your answers in a LaTeX document and submit both a pdf and the LaTeX source. Try to keep a similar template to the one given. You may find it helpful to use sharelatex or install a LaTeX distribution and editor, such as texworks. Some documentation on LaTeX can be found at https://en.wikibooks.org/wiki/LaTeX.

Study material

- Chua’s Circuit
  https://en.wikipedia.org/wiki/Chua\%27s\_circuit

- Pendulum
  https://en.wikipedia.org/wiki/Pendulum\_(mathematics)

- Double Pendulum

- Runge Kutta Methods
  https://en.wikipedia.org/wiki/List\_of\_Runge\%E2\%80\%93Kutta\_methods

- LaTeX
  https://en.wikibooks.org/wiki/LaTeX

- Monte Carlo integration
  https://en.wikibooks.org/wiki/Parallel_Spectral_Numerical_Methods/Introduction\_to_Parallel_Programming#Monte_Carlo_Integration
  https://bitbucket.org/bkmbitbucket/parallelintrorocket/src

- Scientific Computing and Simulation Science - Chapter 1 in:
  https://doi.org/10.1017/CBO9780511812583

- Ordinary differential equations - Sections 4.1 to 4.1.2.3 in:
  http://pages.tacc.utexas.edu/~eijkhout/istc/istc.html

Algorithms

Homework question 1

After reading chapter 1 of Karniadakis and Kirby, review one of the papers on the top 10 Algorithms from the twentieth century. Explain why you think the algorithm is important and how it affects your life.

ANSWER

COMMENT

GRADE

1
Homework question 2


and other material you may find useful, explain what integration is, and how Monte carlo integration works. Write a serial Monte Carlo integration program similar to the examples in Fortran and Python, but in another language you are familiar with and check that it gives the correct answer.

ANSWER
COMMENT
GRADE

Homework question 3

a) Explain how differentiation is the opposite of integration.

ANSWER
COMMENT
GRADE

b) Explain how you might solve

\[ \frac{dy}{dt} = y \quad y(0) = 0.1 \quad t \in [0, 5] \]

using forward Euler method. Write a program that does this.

ANSWER
COMMENT
GRADE

c) Explain how you might solve

\[ \frac{dy}{dt} = y \quad y(0) = 0.1 \quad t \in [0, 5] \]

using backward Euler method. Write a program that does this.

ANSWER
COMMENT
GRADE

d) Explain how you might solve

\[ \frac{dy}{dt} = y \quad y(0) = 0.1 \quad t \in [0, 5] \]

using implicit midpoint rule. Write a program that does this.

ANSWER
COMMENT
GRADE

e) Explain how you might solve

\[ \frac{dy}{dt} = y \quad y(0) = 0.1 \quad t \in [0, 5] \]

using the classical fourth order Runge Kutta method. Write a program that does this.

ANSWER
COMMENT
GRADE
Pendulum

Homework question 4

a) What is the nonlinear pendulum equation and its linearized small angle approximation?

b) Write programs to solve the nonlinear pendulum equation and its linearized approximation. Use both the classic fourth order Runge Kutta method and the implicit midpoint rule. Add plots of the resulting solutions. You are encouraged to also use a programming language that will work on the Rocket cluster.
Chua’s circuit

Homework question 5

a) What is Chua’s circuit?
   ANSWER
   COMMENT
   GRADE

b) Give parameters for which Chua’s circuit has chaotic solutions.
   ANSWER
   COMMENT
   GRADE

c) Write programs to simulate chaotic solutions to Chua’s circuit. Use both the classic fourth order Runge Kutta method and the implicit midpoint rule. Add plots of the resulting solutions. You are encouraged to also use a programming language that will work on the Rocket cluster.
   ANSWER
   COMMENT
   GRADE

d) Describe how you might experimentally realize Chua’s circuit using materials you can find easily, for example at a local electronics store such as Oomipood or similar.
   ANSWER
   COMMENT
   GRADE
Statistics - part of hw 2, but can start early

Homework question 6

a) Compare the long term mean position and standard deviations for the linearized pendulum to those calculated using the implicit midpoint rule and the fourth order Runge Kutta method over long times.

ANSWER
COMMENT
GRADE

b) For different choices of the initial energy, Perform multiple long time simulations of the linear and nonlinear pendulums and calculate the mean square difference between the two numerical solutions. Choose a small enough time step to get accurate initial results. For what level of initial energy are the two models the same to within 90%, 95% and 99% confidence?

ANSWER
COMMENT
GRADE

c) Do the central limit theorem and strong law of large numbers apply to chaotic solutions of Chua’s circuit? Support your answer with numerical evidence for an ensemble of different initial conditions but the same physical parameters. Examine both averages in time for single trajectories and averages at a single point in time, for an ensemble of different initial conditions.

ANSWER
COMMENT
GRADE

d) Estimate joint probability distributions for the physically simulated variables in Chua’s circuit. Do this for two different choices of physical constants and then explain whether you think you can relate the probability distributions you obtain, for example by a mathematical transformation.

ANSWER
COMMENT
GRADE
Bibliography


[9] LaTeX website http://www.latex-project.org/
