MTAT.07.017
Applied Cryptography

Public Key Cryptography
(Asymmetric Cryptography)

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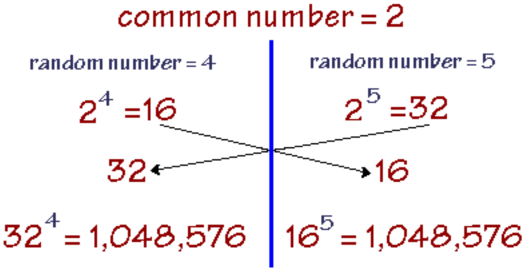
Diffie-Hellman Key Exchange

Ralph Merkle, Martin Hellman, Whitfield Diffie (1976)

- The first public-key algorithm

Diffie-Hellman (DH) Key Exchange



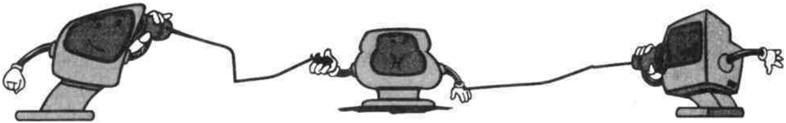
- $(2^5)^4 = 2^{5 \cdot 4} = (2^4)^5$
- In practice multiplicative group of integers modulo p is used
- Discrete logarithm problem:
  - hard to find x, given $2^x = 32 \mod p$
- ElGamal and DSA based on DL problem
- Secure against passive eavesdropping

Adversary (Threat) Model

- What are the capabilities of the adversary?

Passive attacks (eavesdropping):

Active attacks (man-in-the-middle):



- Which attack is harder to execute?
- Without a threat model the word "secure" tells nothing

RSA

Adi Shamir, Ron Rivest, Leonard Adleman (1977)

- The most popular public-key cryptosystem

RSA algorithm

- Key generation:
  1. Choose two distinct prime numbers p and q (usually 1024-bits)
  2. Compute $n = pq$ (2048-bits)
  3. Compute $\varphi(n) = (p-1)(q-1)$
  4. Choose an integer e such that e and $\varphi(n)$ are coprime
  5. Find an integer d such that $de \equiv 1 \mod \varphi(n)$

  n - modulus

  e - public exponent (**e**ncryption exponent)

  d - private exponent (**d**ecryption exponent)

  Public key: (n, e)

  Private key: (d)
- Encryption:
  - $c \equiv m^e \mod n$
- Decryption:
  - $m \equiv c^d \mod n$
- Integer factorization problem

Naive approach:

```
>>> m**e % n
```

Much faster:

```
>>> pow(m, e, n)
```

RSA Encryption

The basis:

What is encrypted with one key can be decrypted only with the another and vice versa.

- What is encryption for?
  - Only the recipient could decrypt
- How do you encrypt?
  - Using public key of the recipient
- How does recipient decrypt?
  - Using his private key

RSA Signing

The concept of signing:

Encryption with private key, decryption with public key

- What is signing for?
  - Everyone could authenticate the origin
- How do you sign?
  - Encrypting with your private key
- How do others verify?
  - Decrypting with your public key

In practice message digest is encrypted (signed)

Public Key Cryptography

The benefits of asymmetric cryptography:

- Provides possibility for digital signatures
  - Data origin authentication
- Encryption key can be negotiated over non-confidential channel
  - Authenticated channel still required

Exponentiation

Which operation is harder to calculate:

$$x^{16384} \qquad \text{or} \qquad x^{8191} \qquad ?$$

```
>>> bin(16384)
'0b100000000000000'
>>> bin(8191)
'0b1111111111111'
```

Number of multiplications: number of bits + number of "1" bits

Example:

$$x^8 = \overbrace{\underbrace{x \cdot x}_{y} \cdot y}^{z} \cdot z \qquad x^7 = \underbrace{x \cdot x}_{y} \cdot y \cdot y \cdot x$$

RSA Exponents

Key generation:

1. Choose two distinct prime numbers p and q
2. Compute $n = pq$
3. Compute $\varphi(n) = (p-1)(q-1)$
4. Choose an integer e such that e and $\varphi(n)$ are coprime
5. Find an integer d such that $de \equiv 1 \mod \varphi(n)$

- Can we choose e that will provide faster encryption?
  - Yes! $e = 2^{16} + 1 = 65537$ (0b10000000000000001)
  - e = 3 (0b11) may also be used (not recommended)
  - Some implementations use random public exponent
- Can we instead choose d that will provide faster decryption?
  - No! This will leak information about secret d
  - In fact, constant time exponentiation must be used for d

Key Length Recommendations (NIST)

Date	Minimum of Strength	Symmetric Algorithms	Asymmetric	Discrete Logarithm		Elliptique Curve	Hash (A)	Hash (B)
				Key	Group			
2010 (Legacy)	80	2TDEA*	1024	160	1024	160	SHA-1** SHA-224 SHA-256 SHA-384 SHA-512	SHA-1 SHA-224 SHA-256 SHA-384 SHA-512
2011 - 2030	112	3TDEA	2048	224	2048	224	SHA-224 SHA-256 SHA-384 SHA-512	SHA-1 SHA-224 SHA-256 SHA-384 SHA-512
> 2030	128	AES-128	3072	256	3072	256	SHA-256 SHA-384 SHA-512	SHA-1 SHA-224 SHA-256 SHA-384 SHA-512
>> 2030	192	AES-192	7680	384	7680	384	SHA-384 SHA-512	SHA-224 SHA-256 SHA-384 SHA-512
>>> 2030	256	AES-256	15360	512	15360	512	SHA-512	SHA-256 SHA-384 SHA-512

http://www.keylength.com/

RSA private key file format

```
$ openssl genrsa -out priv.pem 1024
Generating RSA private key, 1024 bit long modulus
......................++++++
...........++++++
e is 65537 (0x10001)

$ cat priv.pem
-----BEGIN RSA PRIVATE KEY-----
MIICXQIBAAKBgQDadIy9YfBPIk9Imlq0luDERItfj9FINQn/Gv+q+cHk06RgpphX
6c+sIvVk/bEHmWGfVwCcxMF5tzIeC/ns8sHu1b1IsUXSJwiOArzuxPMBfay5TUu1
6oP/K0lBxYaWa3xMSa+FmaAQsugBcWCYTMOiv+H7YkZdpDRZ++IbfWyX1wIDAQAB
AoGAQLdcZlJYVaktYa3Qh0VXSu2feGzrq/+CeZ+u9CDPbxG/1Z4k7Y5npnAoOIVT
Z5Pp1sF4ffjP9FXwM/SKUsbL6n/TR7U253KzjxzfuBPMayjTMqqHTVDwbcJ0zhdG
emF1s3aZtRmZA8nvrooAQqhr5pfNcL/0iOmjf2+E4St3IxECQQD9rhVTm4NV1prr
f2813zebpqbhzUPCbUK9/FmfZcx1Tg7EX6RP1Jf0aTXQgCL9nGrZhKVraSSdwAZP
4q/obaOfAkEA3HP/hsUoi1m4VGXLW0cD+c5UYNcfJkkmHJi7AAbzE06FFrHyJhLC
9CsB4OVayKbhxmAtN6Djhudltav/oFTgyQJAeysm161OG0NTfLwm9vUmPsCvjjrn
tRbTtRta7/wqTdL2iNECQQCU9ufnB5YxyluORScYQ6ij4vXV5tD8buPgQsRhQ5xa
qfzQvWQap0hR3F4Ojq7GcI1orvQcEgYOLFp7VyueqH56
-----END RSA PRIVATE KEY-----
```

PEM format (BASE64 encoded ASN.1 DER)

```
$ openssl rsa -in priv.pem -outform der -out priv.der
writing RSA key
```

RSA private key file format

```
$ dumpasn1 priv.der
  0 605: SEQUENCE {
  4   1:   INTEGER 0
  7 129:   INTEGER
       :     00 DA 74 8C BD 61 F0 4F 22 4F 48 9A 5A B4 96 E0
       :     D7 ...
139   3:   INTEGER 65537
144 128:   INTEGER
       :     40 B7 5C 66 52 58 55 A9 2D 61 AD D0 87 45 57 4A
       :     97 ...
275  65:   INTEGER
       :     00 FD AE 15 53 9B 83 55 D6 9A EB 7F 6F 35 DF 37
       :     9F ...
342  65:   INTEGER
       :     00 DC 73 FF 86 C5 28 8B 59 B8 54 65 CB 5B 47 03
       :     C9 ...
409  64:   INTEGER
       :     7B 2B 26 D7 AD 4E 1B 43 53 7C BC 26 F6 F5 26 3E
       :     DD ...
475  65:   INTEGER
       :     00 87 56 C7 66 CB 9F 6A 7D 78 46 87 FF E2 57 A4
       :     D1 ...
542  65:   INTEGER
       :     00 94 F6 E7 E7 07 96 31 CA 5B 8E 45 27 18 43 A8
       :     7A ...
       :   }
```

RSA private key file format

```
--
-- Representation of RSA private key with information for
-- the CRT algorithm.
--
RSAPrivateKey ::= SEQUENCE {
    version           Version,
    modulus           INTEGER,  -- n
    publicExponent    INTEGER,  -- e
    privateExponent   INTEGER,  -- d
    prime1            INTEGER,  -- p
    prime2            INTEGER,  -- q
    exponent1         INTEGER,  -- d mod (p-1)
    exponent2         INTEGER,  -- d mod (q-1)
    coefficient       INTEGER,  -- (inverse of q) mod p
    otherPrimeInfos   OtherPrimeInfos OPTIONAL
}
```

http://tools.ietf.org/html/rfc3447

RSA public key file format

```
$ openssl rsa -inform der -in priv.der -pubout -outform der -out pub.der
writing RSA key
$ openssl rsa -inform der -in pub.der -pubin -out pub.pem
writing RSA key
$ dumpasn1 pub.der
  0 159: SEQUENCE {
  3  13:   SEQUENCE {
  5   9:     OBJECT IDENTIFIER rsaEncryption (1 2 840 113549 1 1 1)
 16   0:     NULL
       :     }
 18 141:   BIT STRING, encapsulates {
 22 137:     SEQUENCE {
 25 129:       INTEGER
       :         00 DA 74 8C BD 61 F0 4F 22 4F 48 9A 5A B4 96 E0
       :         C4 44 8B 5F 8F D1 48 35 09 FF 1A FF AA F9 C1 E4
       :         D3 A4 60 A6 98 57 E9 CF AC 22 F5 64 FD B1 07 99
       :         61 9F 57 00 9C C4 C1 79 B7 32 1E 0B F9 EC F2 C1
       :         EE D5 BD 48 B1 45 D2 27 08 8E 02 BC EE C4 F3 01
       :         7D AC B9 4D 4B B5 EA 83 FF 2B 49 41 C5 86 96 6B
       :         7C 4C 49 AF 85 99 A0 10 B2 E8 01 71 60 98 4C C3
       :         A2 BF E1 FB 62 46 5D A4 34 59 FB E2 1B 7D 6C 97
       :         D7
157   3:       INTEGER 65537
       :       }
       :     }
       :   }
```

RSA public key file format

```
SubjectPublicKeyInfo  ::=  SEQUENCE  {
    algorithm  AlgorithmIdentifier,
    subjectPublicKey    BIT STRING ::= RSAPublicKey
}

RSAPublicKey ::= SEQUENCE {
    modulus           INTEGER,  -- n
    publicExponent    INTEGER   -- e
}

AlgorithmIdentifier  ::=  SEQUENCE  {
    algorithm  OBJECT IDENTIFIER ::= rsaEncryption,
    parameters ANY DEFINED BY algorithm OPTIONAL ::= NULL
}

rsaEncryption OBJECT IDENTIFIER ::= { pkcs-1 1 }
```

https://tools.ietf.org/html/rfc3280
https://tools.ietf.org/html/rfc3447

Task: RSA utility – 7p

Implement RSA encryption and signing utility.

```
$ ./rsa.py
Usage:
encrypt <public key file> <plaintext file> <output ciphertext file>
decrypt <private key file> <ciphertext file> <output plaintext file>
sign <private key file> <file to sign> <signature output file>
verify <public key file> <signature file> <file to verify>
```

- Must support private and public keys in PEM and DER format (detect by content; use `.decode('base64')`)
- Halt if requested to encrypt plaintext larger than possible
- Must sign SHA256 hash (DigestInfo) of the file to sign
- Verification must output "Verified OK" / "Verification Failure"
- Encryption and signing according to PKCS#1 v1.5
- Use your own ASN.1 DER encoder and pyasn1
- Use pyasn1 version from Ubuntu packages (not from `pip`) (**!**)

RSA PKCS#1 v1.5

Encryption process:

1. Pad plaintext: `0x00 || 0x02 || PS || 0x00 || D`
   - D – plaintext to encrypt
   - PS (padding string) – at least 8 random bytes (except 0x00)
   - Plaintext must be padded to size of modulus n
2. Convert padded byte string to integer
3. Calculate ciphertext: $c \equiv m^e \mod n$
   - in python: `c = pow(m, e, n)`
4. Convert ciphertext integer to byte string

Decryption process:

1. Convert ciphertext to integer
2. Calculate decryption: $m \equiv c^d \mod n$
3. Convert decrypted integer to byte string
4. Remove padding

`https://tools.ietf.org/html/rfc2313`

RSA PKCS#1 v1.5

Signing process:

1. Construct plaintext (DER DigestInfo of the file to sign)
2. Pad plaintext: `0x00 || 0x01 || PS || 0x00 || D`
   - D – plaintext to sign
   - PS (padding string) – zero or more 0xFF bytes
   - Plaintext must be padded to size of modulus *n*
3. Convert padded byte string to integer
4. Calculate signature: $s \equiv m^d \mod n$
5. Convert signature integer to byte string (**length of** ***n***)

Verification process:

1. Convert signature byte string to integer
2. Calculate decryption: $m \equiv s^e \mod n$
3. Convert decrypted integer to byte string
4. Remove padding to obtain DigestInfo DER structure
5. Compare DigestInfo with DigestInfo of the signed file

`https://tools.ietf.org/html/rfc2313`

Task: Test case

```
#!/bin/bash
echo "[+] Generating RSA key pair..."
openssl genrsa -out priv.pem 1017
openssl rsa -in priv.pem -pubout -out pub.pem

echo "[+] Testing encryption..."
echo "hello" > plain.txt
./rsa.py encrypt pub.pem plain.txt enc.txt
openssl rsautl -decrypt -inkey priv.pem -in enc.txt -out dec.txt
diff -u plain.txt dec.txt

echo "[+] Testing decryption..."
openssl rsautl -encrypt -pubin -inkey pub.pem -in plain.txt -out enc.txt
./rsa.py decrypt priv.pem enc.txt dec.txt
diff -u plain.txt dec.txt

echo "[+] Testing signing..."
dd if=/dev/urandom of=filetosign bs=1M count=1
./rsa.py sign priv.pem filetosign signature
openssl dgst -sha256 -verify pub.pem -signature signature filetosign

echo "[+] Testing successful verification..."
openssl dgst -sha256 -sign priv.pem -out signature filetosign
./rsa.py verify pub.pem signature filetosign

echo "[+] Testing failed verification..."
openssl dgst -md5 -sign priv.pem -out signature filetosign
./rsa.py verify pub.pem signature filetosign
Verification Failure
```

Hybrid Encryption

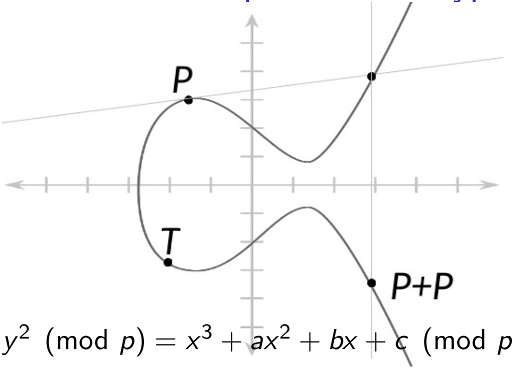
How to encrypt plaintexts larger than modulus size?

- Increase modulus size
  - 2x modulus size increase – 8x slowdown
- Split in blocks and encrypt separately
  - Asymmetric encryption much slower than symmetric
- Use RSA to encrypt symmetric data encryption key:

Questions

- What does public key cryptography give us?
- How will you create RSA encrypted message to me?
- How will you verify my RSA signed message?
- What is hybrid encryption useful for?
- How are passive attacks different from active attacks?
- Why active attacks are harder to execute?
- What is threat model useful for?
- Why 2048-bit RSA does not have security level of 2048-bits?
- What will happen to cryptography the day quantum computers are invented?

Elliptic Curve Cryptography



$y^2 \pmod p = x^3 + ax^2 + bx + c \pmod p$

- Curve defined by a, b, c, p
- Formulas for point addition, doubling
- DL problem: find x, given $a^x = b \mod p$
- EC DL problem: find x, given $\underbrace{P + P + \cdots + P}_{x \ times} = xP = T$
- Security: 256-bit ECC $\approx$ 3072-bit RSA

Estonian ID card (Infineon RSA keygen) flaw

https://news.err.ee/616732/potential-security-risk-could-affect-750-000-estonian-id-cards

Candidate prime p constructed by:

$$p = k \times M + (65537^a \bmod M)$$

- $M = 2 \times 3 \times 5 \times 7 \times 11 \times \ldots \times 701$ (971-bit constant)
- k – 53-bit random number
- a – 255-bit random number

Researchers found how to factor such N using $2^{34.29}$ operations

- Each operation takes 212 ms on Intel Xeon E5-2650 v3 CPU

Moral: Do not invent your own crypto algorithms!