MTAT.05.128: Introduction to Digital Communications **University of Tartu**

Homework assignment 3

Due date: April 19, 2016

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1. Recall that the Hilbert transform of a signal $x(t)$ is defined as

$$\mathcal{H}\left\{x(t)\right\} = x(t) \star \frac{1}{\pi t} = \frac{1}{\pi}\int_{\tau=-\infty}^{\infty} \frac{x(\tau)}{t-\tau} d\tau = \frac{1}{\pi}\int_{\tau=-\infty}^{\infty} \frac{x(t-\tau)}{\tau} d\tau\ .$$

   Prove that the Hilbert transform of a constant is zero. Namely, if $x(t) = c$, then $\mathcal{H}\left\{x(t)\right\} = 0$.

2. Let $m(t)$ be a message signal, which is bandlimited to the range of $(-w, w)$ Hz. In this question, we modify the double sideband with suppressed carrier (DSB-SC) modulation by generating a modulated signal $u(t) = A_c \cdot m(t) \cdot \cos^2(2\pi f_c t)$, where A_c is a constant and $w \ll f_c$ (note that for the carrier we now use $\cos^2(\cdot)$ instead of $\cos(\cdot)$).

   (a) Suggest a demodulation scheme for this modulation and prove mathematically that it allows to recover the original signal.

   (b) What is the minimum usable value of f_c (as a function of w)?

   (c) Demonstrate your demodulation scheme in MATLAB/OCTAVE. Show several examples of signals, their modulated version in time and frequency domain, and the corresponding demodulated signals.

   Hint: $\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$.