MTAT.05.128: Introduction to Digital Communications **University of Tartu**

Homework assignment 1

Due date: March 15, 2016

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1. Reminder (Taylor series): infinitely differentiable at point x_0 function $f(x)$ can be written as

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n \ ,$$

where $f^{(n)}(x_0)$ denotes the n-th derivative of $f(x)$ evaluated at point x_0.

(a) Write the Taylor expansion of the following functions:

$$\text{(i) } \mathsf{e}^x \qquad \text{(ii) } \cos x \qquad \text{(iii) } \sin x \ .$$

(b) Let $\mathsf{e} = 2.718...$ be the natural logarithm base and $i = \sqrt{-1}$. By using the result of (a), show that:

(i) $\mathsf{e}^{ix} = \cos x + i \sin x$;

(ii) $\mathsf{e}^{-ix} = \cos x - i \sin x$;

(iii) is e^x an odd function? is it an even function?

2. Consider the function of period T given by

$$f(t) = t \qquad \text{for} \qquad 0 < t < T \ .$$

(a) Draw this function.

(b) Show that the coefficients in Representation 1 (see reminder at the end) of this function are given by

$$a_0 = \frac{T}{2} \ , \qquad a_n = 0 \text{ for } n \neq 0 \ , \qquad \text{and} \qquad b_n = -\frac{T}{\pi n} \text{ for } n \neq 0 \ .$$

Hints:

– Recall that, due to symmetry,

$$\int_0^{2\pi} \cos x dx = \int_0^{2\pi} \sin x dx = 0 \ .$$

– You can use integration by parts. Thus, for example,

$$\int_0^T x \cos x dx = x \sin x \Big|_0^T - \int_0^T \sin x dx \ .$$

(c) Find the coefficients c_0 and c_n in Representation 2 of the function $f(t)$.

(d) Find the coefficients x_n in Representation 3 of the function $f(t)$.

(e) For one of the representations (of your choice) write a MATLAB code to illustrate that the obtained Fourier series converges to the function $f(t)$ (similarly to what was shown in the class). Show the results for series with 5, 15, 50 and 300 terms. Attach the MATLAB code, and the resulting plots.

Reminder: Fourier series

$f(t)$ is a function of period T, $f(t) = f(t+nT)$ for $n \in \mathbb{Z}$. The fundamental frequency $f_0 = 1/T$.

Representation 1

$$f(t) = a_0 + \sum_{n=1}^{\infty} \Big(a_n \cos(2\pi n f_0 t) + b_n \sin(2\pi n f_0 t) \Big) ,$$

where

$$a_0 = \frac{1}{T} \int_0^T f(t) dt ,$$

$$a_n = \frac{2}{T} \int_0^T f(t) \cos(2\pi n f_0 t) dt \quad \text{for } n \neq 0 \quad ,$$

$$b_n = \frac{2}{T} \int_0^T f(t) \sin(2\pi n f_0 t) dt \quad \text{for } n \neq 0 \quad .$$

Representation 2

$$f(t) = c_0 + \sum_{n=1}^{\infty} \Big(c_n \cos(2\pi n f_0 t + \Theta_n) \Big) ,$$

where

$$c_0 = a_0 , \quad c_n = \sqrt{a_n^2 + b_n^2} \qquad \text{and} \qquad \Theta_n = \arctan\left(-\frac{b_n}{a_n} \right) .$$

Representation 3

$$f(t) = \sum_{n=-\infty}^{\infty} \Big(x_n \cdot \mathsf{e}^{i2\pi n f_0 t} \Big) ,$$

where

$$x_n = \frac{1}{T} \int_0^T f(t) \mathsf{e}^{-i2\pi n f_0 t} dt .$$