MTAT.05.128: Introduction to Digital Communications **University of Tartu**

Final exam

Instructor: Vitaly Skachek May 31st, 2016

Student name: ______________________________

Student ID: ______________________________

1. This exam contains 10 pages. Check that no pages are missing.
2. It is possible to collect up to 120 points. Try to collect as many points as possible.
3. Justify all your answers (where applicable).
4. All facts and results that were proved or stated in the class can be used in your solution without a proof. Such results need to be rigorously formulated.
5. Any printed and written material is allowed in the class. No electronic devices are allowed.
6. Exam duration is 3 hours.
7. Good luck!

Question 1	
Question 2	
Question 3	
Question 4	
Total	

Question 1 (30 points). Consider the periodic function $f(t)$ of period T given by

$$f(t) = \begin{cases} 1 & \text{if} \quad 0 \leq t < T/2 \\ 0 & \text{if} \quad T/2 \leq t < T \end{cases}.$$

(a) Determine its complex Fourier series (you can use any of the three representations).

(b) Determine whether $f(t)$ is a power signal, an energy signal, or neither.

Question 2 (30 points).

Let $m(t)$ be a message signal, which is bandlimited to the range of $(-w, w)$ Hz. Considered a modulated signal $u(t) = A_c \cdot m(t) \cdot \cos(2\pi f_0 t) \cdot \cos(4\pi f_0 t)$, where A_c is a constant and $w \ll f_0$.

(a) Suggest a demodulation scheme for this modulation and show mathematically that it allows to recover the original signal.

(b) What is the minimum usable value of f_0 (as a function of w)?

Reminder: $\cos(x)\cos(y) = \frac{1}{2}(\cos(x-y) + \cos(x+y))$.

Question 3 (30 points).

Consider the following two signals defined over $t \in [0, 2]$:

$$\psi_1(t) = \begin{cases} \frac{\sqrt{3}}{2} & \text{if } 0 \le t \le 1 \\ \frac{1}{2} & \text{if } 1 < t \le 2 \end{cases} \qquad \text{and} \qquad \psi_2(t) = \begin{cases} \frac{1}{2} & \text{if } 0 \le t \le 1 \\ -\frac{\sqrt{3}}{2} & \text{if } 1 < t \le 2 \end{cases} .$$

(a) Show that the signals $\psi_1(t)$ and $\psi_2(t)$ are orthonormal.

(b) Express the signal

$$s(t) = \begin{cases} 0 & \text{if } 0 \le t \le 1 \\ 1 & \text{if } 1 < t \le 2 \end{cases}$$

as a weighted linear combination of $\psi_1(t)$ and $\psi_2(t)$. Determine the weighting coefficients.

Question 4 (30 points).

Consider a BPSK-modulated signal transmitted through the AWGN channel with noise spectral density $N_0/2 = \sigma^2$. Let the constellation points of the transmitted signals be $s_1 = -1$ and $s_2 = +1$, and the received signal be r.

In this question we consider the decision error probability when the probabilities of the transmitted signals are

$$\mathsf{Pr}(s_1) = 0.8 \quad \text{and} \quad \mathsf{Pr}(s_2) = 0.2 \ .$$

(a) Assume that the decision regions are chosen according to the following rules:

$$\begin{cases} \text{if } r < 0 \text{ then the decision is } s_1 \\ \text{if } r \geq 0 \text{ then the decision is } s_2 \end{cases} .$$

Show that in this case, similarly to the symmetric case, the probability of error is given by

$$\mathsf{Pr}(\text{err}) = \mathcal{Q}(\sqrt{2E_s/N_0}) \ ,$$

where $\mathcal{Q}(\cdot)$ is the $\mathcal{Q}$-function.

(b) Suggest how to modify the decision regions such that the decision error decreases.

(c) What is the optimal choice of the decision regions? There is no need to find the boundary explicitly, but you need to explain how the boundary can be found.