Problem 1
Assume that the 2nd order prediction with coefficients $a_1 = 0.81$ and $a_2 = 0.53$ is applied to the sequence of speech samples:
654, 652, 560, 364, 225, 112, 52, 5, -21, -93
1. Write down equation for the prediction filter
2. Write down the transfer function of the prediction and synthesis filters
3. Find the sequence of prediction errors
4. Scalar uniformly quantize the prediction coefficients with step=0.01
5. Scalar uniformly quantize the sequence of prediction errors with step=5
6. Reconstruct the prediction coefficients from the quantized values
7. Reconstruct the sequence of prediction errors from the quantized values
8. Reconstruct the input sequence of speech samples

Solution.
1. The prediction filter is determined by the equation
   \[ e(n) = x(n) - 0.81x(n-1) - 0.53x(n-2) \]
   The synthesis filter is determined by the equation
   \[ x(n) = e(n) + 0.81x(n-1) + 0.53x(n-2) \]
2. The transfer functions of the prediction and synthesis filters are
   \[ H_p(z) = \frac{E(z)}{X(z)} = 1 - 0.81z^{-1} - 0.53z^{-2}, H_s(z) = \frac{1}{1 - 0.81z^{-1} - 0.53z^{-2}} \]
3. The sequence of the prediction errors is:
   654, 122.26, 314.74, 435.16, 366.64, -263.17, -157.97, -96.48, -52.6, -78.64
4. The quantized prediction coefficients are 81 and 53.
5. The quantized sequence of prediction errors is
   131, 24, -63, -87, -73, -53, -32, -19, -11, -16
6. The reconstructed prediction coefficients are 0.81 and 0.53
7. The reconstructed sequence of prediction errors is
   655, 120, -315, -435, -365, -265, -160, -95, -55, -80
8. The reconstructed input sequence of speech samples is
   655.0, 650.5, 559.1, 362.6, 225.1, 109.5, 48.0, 1.92, -28.0, -101.7

Problem 2
The encoded according to the JPEG standard 8×8 block of DCT coefficients is
5, 1110,
(0,2)01, (0,1)0, (0,2)01, (0,2)10, (1,1)1, (0,1)0, (0,1)0, (2,1)0, (0,1)1, (1,1)1, (0,1)0,
(3,1)1, (5,1)0, (0,1)1, (13,1)0, (0,1)1, (2,1)0, (0,1)1, (6,1)1, EOB
1. Reconstruct the block of DCT coefficients (assume that the DC coefficient of the previous block is equal to 101)
2. Find coefficients of the 2-D discrete cosine transform for 8×8 block whose pixel values are all the same and equal to 10. Explain the result.

**Solution.**

1. The block of DCT coefficients has the form

   
   \[
   \begin{bmatrix}
   115 & -9 & 0 & 1 & 1 & -1 & 0 & 0 \\
   -1 & 2 & -1 & 0 & 0 & 1 & 0 & 0 \\
   -2 & -1 & 1 & 0 & -1 & 0 & 1 & 0 \\
   0 & -1 & 0 & 0 & 0 & -1 & -1 & 0 \\
   0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\
   0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
   0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
   0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 
   \end{bmatrix}
   \]

2. The DC coefficient of the transformed block is equal to 80 and all other coefficients are equal to zero. The DC coefficient represents the amplitude of the constant basis function. Since the considered block coincides with the constant basis function and the 2-D DCT is an orthogonal transform we obtain that only DC coefficient is nonzero.

**Problem 3**

Let decimated low-frequency and high-frequency parts of the input sequence be equal to

\[
\begin{bmatrix}
43 & 44 & 27 \\
-26 & -13 & -9 
\end{bmatrix}
\]

respectively.

1. Reconstruct the input signal by using the following synthesis filter bank

   \[
g_0(n) = [-1,4,8,2]/\sqrt{85} \text{ (pulse response of the low-pass filter)},
   \]

   \[
g_1(n) = 2,[-8,4,1]/\sqrt{85} \text{ (pulse response of the high-pass filter)}
   \]
Hint:
- perform cyclic extension by placing the last samples to the beginning of the sequence
- normalize the output sequence after reconstruction

2. Write down the frequency response of the low-pass filter.

Solution.
The upsampled reference and detail sequences are

\[
\begin{array}{llllllllll}
43 & 0 & 44 & 0 & 27 & 0 \\
-26 & 0 & -13 & 0 & -9 & 0 \\
\end{array}
\]

After cyclic extension we obtain

\[
\begin{array}{llllllllll}
0 & 27 & 0 & 43 & 0 & 44 & 0 & 27 & 0 \\
0 & -9 & 0 & -26 & 0 & -13 & 0 & -9 & 0 \\
\end{array}
\]

The reconstructed low-frequency part is

\[
\begin{array}{llllllllllllll}
0 & -27 & 108 & 173 & 226 & 300 & 262 & 325 & 196 & 216 & 54 & 0 \\
\end{array}
\]

The reconstructed high-frequency part is

\[
\begin{array}{llllllllllllll}
0 & -18 & 72 & -88 & 199 & -130 & 78 & -70 & 59 & -36 & -9 & 0 \\
\end{array}
\]

Sum of the low-frequency and high-frequency parts is

\[
\begin{array}{llllllllllllll}
0 & -45 & 180 & 85 & 425 & 170 & 340 & 255 & 255 & 180 & 45 & 0 \\
\end{array}
\]

After normalization and windowing we obtain that the reconstructed sequence is

\[
\begin{array}{llllllllllllll}
1 & 5 & 2 & 4 & 3 & 3 \\
\end{array}
\]

The frequency response of the low-pass filter is

\[
H(e^{j\omega T_s}) = (-1 + 4e^{-j\omega T_s} + 8e^{-j2\omega T_s} + 2e^{-j4\omega T_s}) / \sqrt{85}
\]