MTAT.05.125: Introduction to Theoretical Computer Science **University of Tartu**

Midterm exam

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Student name: _____

Student ID: _____

1. This exam contains 10 pages. Check that no pages are missing.
2. It is possible to collect up to 110 points. Try to collect as many points as possible.
3. Justify and prove all your answers (where applicable).
4. All facts and results that were proved or stated in the class can be used in your solution without a proof. Such results need to be rigorously formulated.
5. Any printed and written material is allowed in the class. No electronic devices are allowed.
6. Exam duration is 1 hour 40 minutes.
7. Good luck!

Question 1	
Question 2	
Question 3	
Question 4	
Total	

Question 1 (46 points).

A student came to a library. There are 100 different book titles in the library. (All books of the same title are assumed to be identical.)

(a) If the student is to choose five *different* books, how many choices he/she has?

A group of 15 students from the Institute of Computer Science came to the library, and together they took 5 copies of "Introduction to Combinatorics", 7 copies of "Introduction to the Theory of Computation", and 3 copies of "How To Do Well In Exams".

(b) How many ways to distribute the books to the students are there, if each student got exactly one book?

"Harry Potter and the Order of the Phoenix" is a very popular book. A group of 10 students came to the library, and together they took 25 copies of "Harry Potter and the Order of the Phoenix".

(c) How many ways to distribute the copies of the book to the students are there?

(d) Similar to (c), but additionally, each student got at least one copy of the book. How many ways to distribute the copies of the book to the students are there?

(e) Similar to (c), but additionally, no student got more than 10 copies of the book. How many ways to distribute the copies of the book to the students are there?

Question 2 (16 points).

By using the method that was shown in the class, construct a non-deterministic automaton, which accepts the language that is described by the following regular expression, where $\Sigma = \{0, 1, 2\}$:

$$(0^* \cup 1^* \cup 2^*)(012) \; .$$

Show all intermediate steps in the algorithm.

Question 3 (24 points).

Definition. A prefix of a string w is a substring that occurs at the beginning of w.

Example. The substrings ε, 0, 00 and 001 are all prefixes of the string 001. By contrast, 1 is not a prefix of 001.

Let $\Sigma = \{0, 1\}$ be an alphabet.

(a) Let $\mathcal{R}$ be a regular language. Is the following language regular or not? Justify your answer.

$$\mathcal{L}_R = \left\{ w \;\middle|\; w \in \{0,1\}^* \text{ is a string that contains a prefix } z \in \mathcal{R} \right\}.$$

(b) Let $\mathcal{N}$ be a non-regular language. Is the following language always non-regular? Justify your answer.

$$\mathcal{L}_N = \left\{ w \;\middle|\; w \in \{0,1\}^* \text{ is a string that contains a prefix } z \in \mathcal{N} \right\}.$$

Question 4 (24 points).

Let $\Sigma = \{0, 1, \#\}$ be an alphabet.

(a) Is the following language regular? Justify your answer.

$$\mathcal{L}_1 = \left\{ w\#a \;\middle|\; w \in \{0,1\}^*, a \in \{0,1\},\ w \text{ is a string that starts with a symbol } a \right\}.$$

(b) Prove that the following language is not regular:

$$\mathcal{L}_2 = \left\{ w\#z \;\middle|\; w, z \in \{0,1\}^* \text{ are two strings, and } z \text{ is a prefix of } w \right\}.$$