

Midterm exam – second retake

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Student name: _____

Student ID: _____

1. This exam contains 10 pages. Check that no pages are missing.
2. It is possible to collect up to 120 points. Try to collect as many points as possible.
3. Justify and prove all your answers (where applicable).
4. All facts and results that were proved or stated in the class can be used in your solution without a proof. Such results need to be rigorously formulated.
5. Any printed and written material is allowed in the class. No electronic devices are allowed.
6. Exam duration is 1 hour 40 minutes.
7. Good luck!

Question 1	
Question 2	
Question 3	
Question 4	
Total	

Question 1 (48 points).

How many different words can be composed from the letters

A, U, T, O, M, A, T, A

under the following conditions?

(Each letter in the list should be used exactly once. The word does not have to be a legal word in any human language. For example, “TUOMATAA” is a legal word, but “AUTO” is not).

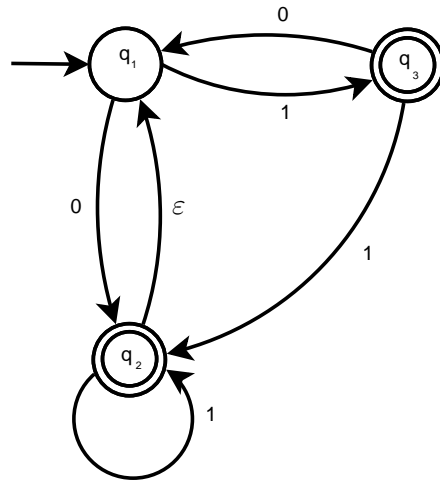
- (a) No restrictions.
- (b) The word starts with letter “U” and ends with “MA”.
- (c) The subsequence “TT” does not appear in the word.
- (d) The subsequences “TT”, “MU” and “MO” do not appear in the word.

Now, assume that you can use an unlimited number of each of the letters A, U, O, M, T.

- (e) How many words of length 8 (without restrictions) can you compose?

Question 2 (24 points).

Convert the following nondeterministic finite automaton into an equivalent deterministic automaton. Show all the steps in the conversion process.



Question 3 (24 points).

By using the method that was shown in the class, construct a non-deterministic automaton, which accepts the language that is described by the following regular expression:

$$((00) \cup (101))^* .$$

Show all intermediate steps in the algorithm.

Question 4 (24 points).

(a) Prove that the following language is not regular:

$$\mathcal{L}_1 = \{ 0^n 1^m \mid 3n \leq m \leq 5n + 2, n, m \in \mathbb{N} \} .$$

(b) Is the following language regular or not? Justify your answer.

$$\mathcal{L}_2 = \{ w \mid w \text{ is a string with } n \text{ zeros and } m \text{ ones, where } 3n \leq m \leq 5n + 2, n, m \in \mathbb{N} \} .$$

