Homework Assignment 7

Due date: December 22nd, 2016

It is possible to collect up to 110 points in this homework.

1. Answer whether $f(n) = O(g(n))$ and whether $f(n) = o(g(n))$ (justify your answers):
   (a) $f(n) = 2n + \log_2 n$ and $g(n) = n$;
   (b) $f(n) = \log_2 n$ and $g(n) = n^{\frac{1}{10}}$;
   (c) $f(n) = n^2$ and $g(n) = 2^n$.

2. **Definition:** a *degree* of a vertex v in an undirected graph is the number of edges incident to v, when self-loops are counted as two edges.

   Define a language DEGREE-k:

   $$\text{DEGREE-}k = \{\langle G, k\rangle \mid G \text{ is an undirected graph that contains a vertex of degree } k\} \ .$$

   Prove that DEGREE-$k \in \mathcal{P}$.

3. Define a language DOUBLE-SAT:

   $$\text{DOUBLE-SAT} = \{\langle \phi\rangle \mid \phi \text{ is a CNF-formula that has at least two } \textbf{different} \text{ satisfying assignments}\} \ .$$

   Prove that DOUBLE-SAT is $\mathcal{NP}$-complete.

   Hint: first, show that DOUBLE-SAT $\in \mathcal{NP}$. Second, show a polynomial-time reduction from SAT to DOUBLE-SAT.

4. **Definition:** A *disconnected set* $\mathcal{S}$ in a graph G is a set of vertices such that for every pair of vertices u, v in G, if there is no edge between v and u, then at least one of these two vertices is in $\mathcal{S}$.

   Define a language DISCONNECTED-SET:

   $$\text{DISCONNECTED-SET} = \big\{\langle G, k\rangle \mid G \text{ is an undirected graph that contains a disconnected set of size } k\big\} \ .$$

   Prove that DISCONNECTED-SET is $\mathcal{NP}$-complete.

   Hint: first, show that DISCONNECTED-SET $\in \mathcal{NP}$. Second, show a polynomial-time reduction from CLIQUE to DISCONNECTED-SET.