Homework 6

Due date: December 5th, 2016

It is possible to collect up to 110 points in this homework.

1. Define the language

$$\mathcal{L}_1 = \{\langle A\rangle \mid A \text{ is a DFA and } L(A) = (01)^*\} \ .$$

   Show that $\mathcal{L}_1$ is a decidable language.

2. Define the language

$$\mathcal{L}_3 = \{\langle A\rangle \mid A \text{ is a DFA and } L(A) \text{ contains exactly three strings } w \text{ in } (01)^*\} \ .$$

   Show that $\mathcal{L}_3$ is a decidable language.

3. Let $\mathcal{L}$ be a Turing-recognizable language. Additionally, assume that $\overline{\mathcal{L}} \leq_{\mathrm{M}} \mathcal{L}$, where $\overline{\mathcal{L}}$ is a complementary language of $\mathcal{L}$. Show that $\mathcal{L}$ is Turing-decidable.

4. Define the language

$$\mathcal{L}_4 = \{\langle \mathcal{M}\rangle \mid \mathcal{M} \text{ is a Turing machine and } L(\mathcal{M}) = (01)^*\} \ .$$

   Prove that $\mathcal{L}_4$ is an undecidable language.

   Hint: for example, you can use reduction from the language $\mathcal{L}_{\mathrm{TM}}$. Assume that there exists a Turing machine that decides $\mathcal{L}_4$. Show how to construct a Turing machine that decides $\mathcal{L}_{\mathrm{TM}}$.