Homework 6

Due date: December 5th, 2016

It is possible to collect up to 110 points in this homework.

1. Define the language

\[ \mathcal{L}_1 = \{\langle A \rangle \mid A \text{ is a DFA and } L(A) = (01)^* \} . \]

Show that \( \mathcal{L}_1 \) is a decidable language.

2. Define the language

\[ \mathcal{L}_3 = \{\langle A \rangle \mid A \text{ is a DFA and } L(A) \text{ contains exactly three strings } w \text{ in } (01)^* \} . \]

Show that \( \mathcal{L}_3 \) is a decidable language.

3. Let \( \mathcal{L} \) be a Turing-recognizable language. Additionally, assume that \( \overline{\mathcal{L}} \leq_M \mathcal{L} \), where \( \overline{\mathcal{L}} \) is a complementary language of \( \mathcal{L} \). Show that \( \mathcal{L} \) is Turing-decidable.

4. Define the language

\[ \mathcal{L}_4 = \{\langle M \rangle \mid M \text{ is a Turing machine and } L(M) = (01)^* \} . \]

Prove that \( \mathcal{L}_4 \) is an undecidable language.

Hint: for example, you can use reduction from the language \( \mathcal{L}_{\text{TM}} \). Assume that there exists a Turing machine that decides \( \mathcal{L}_4 \). Show how to construct a Turing machine that decides \( \mathcal{L}_{\text{TM}} \).