MTAT.05.125: Introduction to Theoretical Computer Science **University of Tartu**

Homework Assignment 5

Due date: November 21st, 2016

It is possible to collect up to 110 points in this homework.

1. Give an implementation-level description of the Turing machine that decides the following language over the alphabet $\Sigma = \{0, 1\}$:

$$\mathcal{L} = \{\ w \mid w \text{ contains more zeros than ones}\}\ .$$

2. Let $\mathcal{L}$ be a regular language over the alphabet Σ. Show that there exists a deterministic Turing machine $\mathcal{M}$ that decides $\mathcal{L}$.

3. Show that if $\mathcal{L}_1$ and $\mathcal{L}_2$ are two Turing-decidable languages, then

   (a) Intersection $\mathcal{L}_1 \cap \mathcal{L}_2$ is a Turing-decidable language;

   (b) Concatenation $\mathcal{L}_1 \circ \mathcal{L}_2$ is a Turing-decidable language.

   Hint: assume that there exist Turing machines $\mathcal{M}_1$ and $\mathcal{M}_2$ that decide $\mathcal{L}_1$ and $\mathcal{L}_2$, respectively. Use them to construct two Turing machines that decide the languages $\mathcal{L}_1 \cap \mathcal{L}_2$ and $\mathcal{L}_1 \circ \mathcal{L}_2$, respectively.

4. Define a new type of Turing machine, called "no-left-move Turing machine". It is similar to an ordinary deterministic Turing machine, but the transition function is

$$\delta\ :\ Q \times \Gamma \longrightarrow Q \times \Gamma \times \{\mathsf{R}, \mathsf{S}\}\ .$$

   In no-left-move Turing machine, the head can move to the right (R) or to stay in the same position (S). Show that no-left-move Turing machine is not equivalent to the ordinary Turing machine. More specifically, prove that all the languages recognized by the no-left-move Turing machines are regular.

   Hint: assume that $\mathcal{M}$ is a no-left-move Turing machine. Note that in any computation, the head of $\mathcal{M}$ never moves to the left. Construct a deterministic finite automaton that recognizes the same language as the one recognized by $\mathcal{M}$.