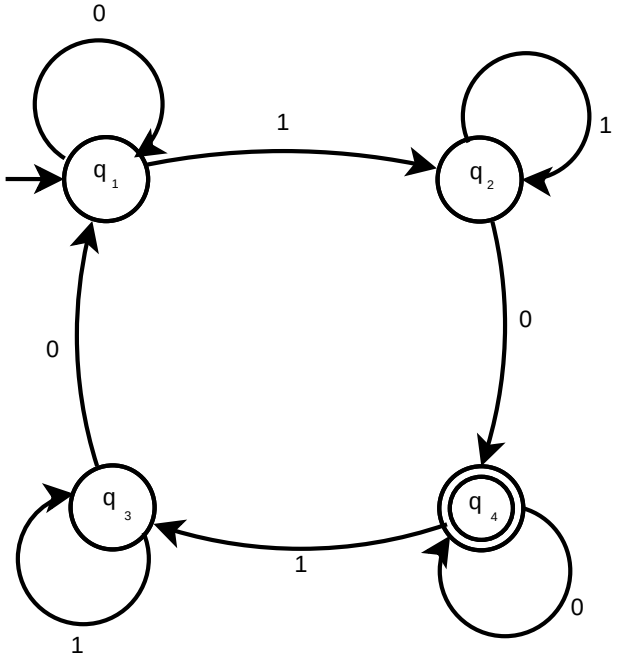
MTAT.05.125: Introduction to Theoretical Computer Science **University of Tartu**

Homework 3

Due date: October 20th, 2016

It is possible to collect up to 110 points in this homework.

1. Describe all the ingredients Q, Σ, δ, q_0 and F of the following deterministic finite automaton $\mathcal{M} = (Q, \Sigma, \delta, q_0, F)$. What language is recongnized by $\mathcal{M}$?



2. Let $\boldsymbol{z} = (y_1, y_2, \cdots, y_m)$ be a given binary prefix, and let $\Sigma = \{0, 1\}$.

   (a) Construct a deterministic finite automaton $\mathcal{M} = (Q, \Sigma, \delta, q_0, F)$, which accepts the language $\mathcal{L}$ that consists of strings that start with the prefix $\boldsymbol{z}$.

   (b) Construct a deterministic finite automaton $\mathcal{M} = (Q, \Sigma, \delta, q_0, F)$, which accepts the language $\mathcal{L}$ that consists of all binary strings that start with any prefix except $\boldsymbol{z}$.

   In both parts describe all the ingredients Q, Σ, δ, q_0 and F.

   Hint: try first to take some specific $\boldsymbol{z}$, for example $\boldsymbol{z} = (001)$, and solve the problem for that $\boldsymbol{z}$. How do you generalize your solution to a general $\boldsymbol{z}$?

3. For two languages $\mathcal{L}_1$ and $\mathcal{L}_2$, define their symmetric difference $\mathcal{L}_1 \oplus \mathcal{L}_2$ as follows:

$$\mathcal{L}_1 \oplus \mathcal{L}_2 = \{\boldsymbol{w} \mid \boldsymbol{w} \text{ belongs to exactly one language } \mathcal{L}_1 \text{ or } \mathcal{L}_2\} \ .$$

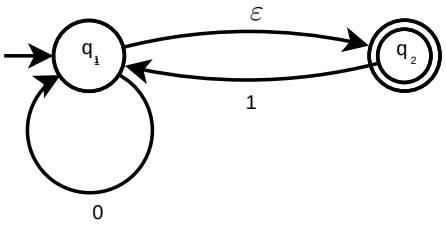
Show that if $\mathcal{L}_1$ and $\mathcal{L}_2$ are two regular languages over the alphabet Σ, then $\mathcal{L}_1 \oplus \mathcal{L}_2$ is regular over the same alphabet.

Example: if $\mathcal{L}_1 = \{001, 110, 11\}$ and $\mathcal{L}_2 = \{110, 00\}$, then $\mathcal{L}_1 \oplus \mathcal{L}_2 = \{001, 11, 00\}$.

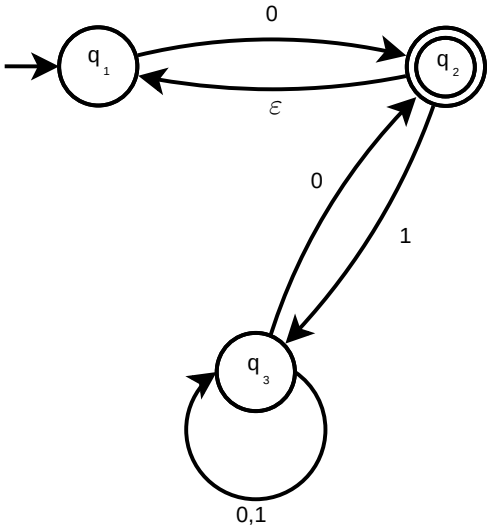
Hint: in the lecture we showed that $\mathcal{L}_1 \cup \mathcal{L}_2$ is regular. Think how to modify that proof.

4. (a) Describe all the ingredients Q, Σ, δ, q_0 and F of each of the following nondeterministic finite automata $\mathcal{N} = (Q, \Sigma, \delta, q_0, F)$. What language is recongnized by each $\mathcal{N}$?

   (b) Convert each of these nondeterministic finite automata into an equivalent deterministic automaton.



(i)



(ii)