Homework 2

Due date: October 6th, 2016

It is possible to collect up to 110 points in this homework.

1. (a) By using the binomial of Newton, show that

$$n(n-1) \cdot (1+x)^{n-2} = \sum_{i=2}^{n} i(i-1)\binom{n}{i} \cdot x^{i-2} .$$

   (b) Conclude that

$$n(n-1) \cdot 2^{n-2} = \sum_{i=2}^{n} i(i-1)\binom{n}{i} .$$

2. In a class of students:

   - 15 students like vanilla ice cream;
   - 18 students like chocolate ice cream;
   - 16 students like strawberry ice cream;
   - 8 students like vanilla and chocolate ice cream;
   - 7 students like vanilla and strawberry ice cream;
   - 8 students like chocolate and strawberry ice cream;
   - 5 students like all three types of ice cream;
   - 4 students do not like any type of ice cream.

   What is the total number of the students in the class?

3. A student rolls 10 identical dice, each dice has six numbers written on its surfaces: '1', '2', '3', '4', '5' and '6'. How many outcomes are possible such that all 6 numbers appear?

4. n gentlemen came to the theater and gave their hats and their jackets to a hatcheck lady. When they leave the theater, no gentleman got both his hat and his jacket. Assume that all hats and jackets are different. In how many ways this can be done?

5. Give a combinatorial proof to the following identity:

$$\sum_{r=0}^{n}(-1)^r\binom{n}{r} \cdot 3^{n-r} = 2^n ,$$

   where n is a positive integer.