

Homework assignment 1

Due date: September 22nd, 2016

1. In the homework in Introduction to Theoretical Computer Science there was the following question: “how many binary vectors of length n with at most m zeros are there?” One student has answered “First, choose $n - m$ places for ones. Then, in the remaining m places put either zero or one. Therefore, the answer is $\binom{n}{n-m} \cdot 2^m$.”
 - (a) Explain what was the mistake in the proposed solution.
 - (b) What is the right answer?
2. How many 6-digit pin-codes are there ...
 - (a) that do not contain digits ‘3’, ‘5’ and ‘6’?
 - (b) that consist of digits ‘2’, ‘2’, ‘4’, ‘4’, ‘4’, ‘7’, and do not contain two consecutive digits ‘2’?

3. Prove that

$$\binom{n}{k} \binom{n-k}{k} \cdot k! = \frac{n!}{k! (n-2k)!},$$

where $n \geq 2k$, in two ways:

- (a) algebraically;
 - (b) by using combinatorial proof. (Hint: how many ways are there to form k pairs out of n people?)
4. How many different words can be composed from the letters

A, U, T, O, M, A, T, A

under the following conditions?

(Each letter in the list should be used exactly once. The word does not have to be a legal word in any human language. For example, “TUOMATAA” is a legal word, but “AUTO” is not).

- (a) No restrictions.
- (b) The word starts with letter “U” and ends with “MA”.
- (c) The subsequence “TT” does not appear in the word.

The assignment continues in the next page...

Now, assume that you can use an unlimited number of each of the letters A, U, O, M, T.

- (d) How many words of length 8 (without restrictions) can you compose?
5. A student rolls five identical dice, each dice has six numbers written on its surfaces: '1', '2', '3', '4', '5' and '6'. How many outcomes are possible ...
- (a) without restrictions?
 - (b) if number '3' appears at least two times?
 - (c) if number '3' appears at most two times?