MTAT.05.125: Introduction to Theoretical Computer Science **University of Tartu**

Final exam

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Student name: ______________________________________

Student ID: ____

1. This exam contains 10 pages. Check that no pages are missing.
2. It is possible to collect up to 120 points. Try to collect as many points as possible.
3. Justify and prove all your answers (where applicable).
4. All facts and results that were proved or stated in the class can be used in your solution without a proof. Such results need to be rigorously formulated.
5. Any printed and written material is allowed in the class. No electronic devices are allowed.
6. Exam duration is 2 hours.
7. Good luck!

Question 1	
Question 2	
Question 3	
Question 4	
Total	

Question 1 (20 points).

Which of the following equations are true or false? Justify your answers.

(a) $n = o(2n + 3)$;

(b) $3n^2 = O(n^3)$;

(c) $n + \log_2 n = O(\log n)$.

Question 2 (35 points).

Define the language

$$\mathcal{L}^\star = \left\{ \langle A \rangle \mid A \text{ is a DFA that accepts no strings of the form } 0\underbrace{11\cdots1}_{n}0 \text{ for } n \geq 2 \right\} .$$

Show that $\mathcal{L}^\star$ is a decidable language.

Hint: one possible way to solve this question is to use the fact that $\mathcal{L}_\varnothing$ is a decidable language (shown in the lecture), where

$$\mathcal{L}_\varnothing = \{\langle A \rangle \mid A \text{ is a DFA and } L(A) = \varnothing\} .$$

Consider a DFA B that accepts all the strings of the form $0\underbrace{11\cdots1}_{n}0$, $n \geq 2$. Why such B exists?

Construct a new automaton, which uses B.

Question 3 (25 points).

Definition: a *pentagon* in an undirected graph $\mathcal{H}$ with five vertices v_1, v_2, v_3, v_4 and v_5, such that four edges connecting v_i with v_{i+1}, $i = 1, 2, 3, 4$, and the edge connecting v_1 with v_5, all appear in $\mathcal{H}$. (In other words, a pentagon is a simple cycle of length five.)

Define a language PENTAGON:

$$\mathsf{PENTAGON} = \{\langle G \rangle \mid G \text{ is an undirected graph that contains a pentagon}\} \ .$$

Is $\mathsf{PENTAGON} \in \mathcal{P}$? Justify your answer.

Question 4 (40 points).

Define a language SAT-7-CLAUSES:

SAT-7-CLAUSES $= \{\langle \phi \rangle \mid \phi$ is a satisfiable CNF-formula where each variable appears in at most 7 clauses$\}$.

In this question, you will show that SAT-7-CLAUSES $\in \mathcal{NP}$-complete.

(a) Prove that SAT-7-CLAUSES $\in \mathcal{NP}$.

(b) Prove that SAT-7-CLAUSES $\in \mathcal{NP}$-hard.
Hint: you can use a polynomial-time reduction from SAT to SAT-7-CLAUSES. The reduction f, for a given CNF-formula ϕ, builds another CNF-formula $f(\phi)$, where each variable appears in at most 7 clauses. Do not forget to show that the reduction is correct and polynomial-time.