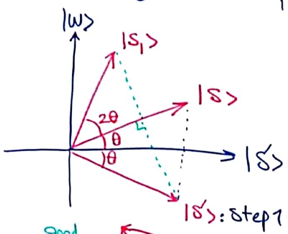
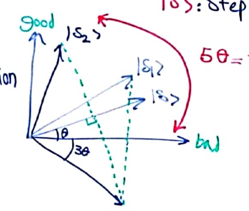
Grover's search algorithm: $f:\{0,1\}^n \longrightarrow \{0,1\}$, $f(\omega)=1$



(1) phase inversion : $\mathbb{1}-2P_1$

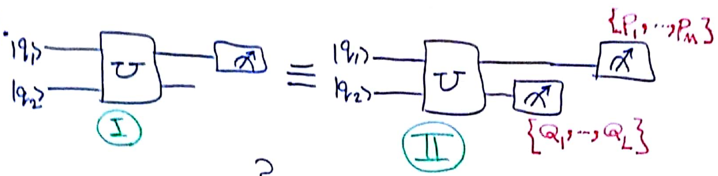
(2) reflection about $|g_1\rangle$

Second iteration



$5\theta = 3\theta + 2\theta \cdots$ iteration t: $(2t+1)\theta$

HW13. Q1.



$$Pr(q_1=m) \overset{?}{=} \sum_{\substack{\ell \\ P(q_2=\ell)\neq 0}} pr(q_1=m \mid q_2=\ell)\, Pr(q_2=\ell) = \sum_{\substack{\ell \\ pr(q_2=\ell)\neq 0}} \frac{pr(q_1=m \cap q_2=\ell)}{pr(q_2=\ell)}\, pr(q_2=\ell)$$

I: Projective measurement $= P_m \otimes \mathbb{1}^{\otimes \ell}$

II: $= P_m \otimes Q_\ell =$ $\quad q_1 = m$, $\quad q_2 = \ell$

$= (P_m \otimes \mathbb{1}^{\otimes \ell})(\mathbb{1}^{\otimes m} \otimes Q_\ell)$ $\qquad \sum_{i=1}^{L} Q_i \,\mathbb{1}$

state to be measured is $|\psi\rangle \longrightarrow$ I: $Pr(q_1=m) = \langle \psi | P_m \otimes \mathbb{1}^{\otimes \ell} | \psi \rangle$

$$= \langle \psi | P_m \otimes \left(\sum_{\ell=1}^{L} Q_\ell\right) | \psi \rangle = \sum_{l=1}^{L} \langle \psi | P_m \otimes Q_l | \psi \rangle$$

$$= \sum_{\ell=1}^{L} Pr(q_1=m, q_2=\ell) = \sum_{\ell} Pr(q_1=m \mid q_2=\ell) \cdot Pr(q_2=\ell)$$

HW13. Q2. QPE, $|I\rangle = |\psi\rangle = \sum_{j=1}^{n} \alpha_j |\psi_{\lambda_j}\rangle$ → linear combination of eigenstates.

(a)

(Lemma p11): $P(|L-\lambda|>\delta \mid \lambda) \le \frac{1}{2(\delta 2^{m}-1)}$

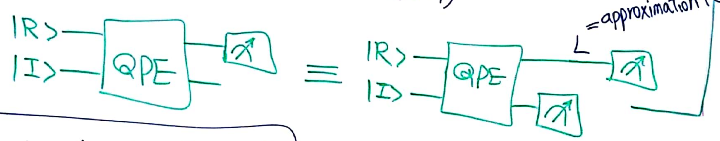
$$P(\forall \lambda, |L-\lambda|>\delta) = \sum_{\lambda} P(|L-\lambda|>\delta \mid \lambda)\, P(\lambda)$$

$$\le \sum_{\lambda} \frac{P(\lambda)}{2(\delta 2^{m}-1)}$$

$$= \frac{1}{2(\delta 2^{m}-1)} \underbrace{\sum_{\lambda} P(\lambda)}_{1}$$

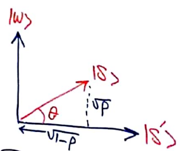
$$= \frac{1}{2(\delta 2^{m}-1)}$$

QPE:



(b) $\boxed{m = b + \log_2\left(2 + \frac{1}{2(1-\varepsilon)}\right)}$

QAE: $\sqrt{P} = \sin\theta$



$W = G(2|0^m\rangle\langle 0^m| - \mathbb{1})G^{\dagger}(\mathbb{1} - 2P_1) = e^{2\pi i H}$

reflection about $|g_1\rangle = G|0^m\rangle$ — the phase inversion

$|S\rangle = \sqrt{p}\,|\omega\rangle + \sqrt{1-p}\,|S'\rangle$

, eigenvalues of W are $e^{\pm 2i\theta}$

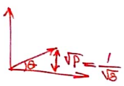
→ eigenvalues of H are ⟶ $e^{\pm 2i\theta} = e^{2\pi i \lambda_H}$ ⟶ $\boxed{\lambda_H = \pm\frac{\theta}{\pi}}$

HW13. Q3. $f: \underbrace{\{0,1\}^3}_{2^3=8} \to \{0,1\}$, $f(x)=\begin{cases} 1 & x=3 \\ 0 & \# \end{cases}$

$x \to \boxed{U_f} \to f(x)$

$$t = \frac{\pi/2 - \arcsin(\sqrt{p})}{2\arcsin(\sqrt{p})} \quad , \sqrt{p} = \frac{1}{\sqrt{8}}$$

MATLAB :
```
>sqP= 1/sqrt(8);
>t=((Pi/2)-asin(sqP))/(2*asin(sqP))
```
$t \simeq 1.6734 \longrightarrow 1 \leq t \leq 2$ $\quad \boxed{\theta = \arcsin\left(\frac{1}{\sqrt{8}}\right)}$



(a) $\lfloor t \rfloor = 1$

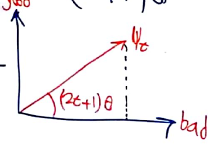
(b) $\lceil t \rceil = 2$ $\longrightarrow$ Pr (success after t iterations) $= \sin^2((2t+1)\theta)$

$t=1 \longrightarrow 0{,}7812 \to 78{,}12\%$

$t=2 \longrightarrow 0{,}9453 \to 94{,}53\%$

angle is $(2t+1)\theta$

$\sqrt{P_t} = \sin((2t+1)\theta) \longleftarrow$



$|\psi_t\rangle = \underset{\downarrow \sqrt{P_t}}{\alpha_{good}} |\psi_{good}\rangle + \alpha_{bad} |\psi_{bad}\rangle$