HW9. Q3. $H = \frac{1}{2}\mathbb{1} + \frac{3}{16} X \otimes Y$, $m=4$, QPE: $|I\rangle := |11\rangle$ with $e^{2\pi i H}$

$$X = \{\overset{+1}{|+\rangle}, \overset{-1}{|-\rangle}\}, \quad Y = \{|\circlearrowright\rangle, |\circlearrowleft\rangle\} \longrightarrow X \otimes Y = \{\overset{|\lambda_{00}\rangle}{|+\circlearrowright\rangle}, \underset{\frac{5}{16}}{\overset{|\lambda_{01}\rangle}{|+\circlearrowleft\rangle}}, \overset{|\lambda_{10}\rangle}{|-\circlearrowright\rangle}, \overset{|\lambda_{11}\rangle}{|-\circlearrowleft\rangle}\} \longrightarrow \text{eigenvectors of } H$$

$$H|+\circlearrowright\rangle = \frac{11}{16}|+\circlearrowright\rangle, \quad H|+\circlearrowleft\rangle = \frac{5}{16}|+\circlearrowleft\rangle, \quad H|-\circlearrowright\rangle = \frac{5}{16}|-\circlearrowright\rangle, \quad H|-\circlearrowleft\rangle = \frac{11}{16}|-\circlearrowleft\rangle \longrightarrow \left\{\frac{5}{16}, \frac{11}{16}\right\} : \lambda \text{ of } H$$

Step 1: $|R, I\rangle = \frac{1}{4}\sum_{x=0}^{15} |x\rangle|11\rangle$

$$\begin{cases} \lambda_0 = \frac{5}{16} \approx 0{,}0100 = \frac{1}{4} = \frac{4}{16} \\ \lambda_1 = \frac{11}{16} = 0{,}1011 \end{cases}$$

Step 2: Loops: $\left(\sum_{t=0}^{m-1} |t\rangle\langle t| \otimes e^{2\pi i H}\right)(|R, I\rangle) = \left[\sum_{t=0}^{15} |t\rangle\langle t| \otimes e^{2\pi i H}\right]\left[\frac{1}{4}\sum_{x=0}^{15} |x\rangle|11\rangle\right]$

$$\boxed{|I\rangle = \frac{|\lambda_{00}\rangle - |\lambda_{10}\rangle - i|\lambda_{11}\rangle - i|\lambda_{01}\rangle}{2}}$$

$$= \frac{1}{4}\sum_{x=0}^{15} |x\rangle e^{2\pi i H}|11\rangle$$

$$= \frac{1}{8}\left[\sum_{x=0}^{15} |x\rangle|\lambda_{00}\rangle e^{2\pi i \lambda_0 x} - \sum_{x=0}^{15} e^{2\pi i \lambda_1 x}|x\rangle|\lambda_{10}\rangle + \cdots\right]$$

Step 3. $\text{QFT}^{\dagger} \longrightarrow |R, I\rangle = \frac{1}{8}\left[\sum_{\xi=0}^{15} \hat{f}(\xi)|\xi\rangle|\lambda_{00}\rangle + \cdots\right]$, $\hat{f}(\xi) = \frac{1}{4}\frac{1-\omega_\xi^{16}}{1-\omega_\xi}$, $\omega_\xi = e^{2\pi i(\lambda - \xi/16)}$

$$= \frac{1}{4.8}\left[\sum_{\xi=0}^{15} \frac{1-e^{2\pi i(\lambda_0 - \xi/16)\cdot 16}}{1-e^{2\pi i(\lambda_0 - \xi/16)}}|\xi\rangle|\lambda_{00}\rangle + \cdots\right] \longrightarrow \xi/16 = \lambda_0 = \frac{4}{16} \longrightarrow \xi = 4$$

$$= \frac{1}{32}\left[16\,|0100\rangle|\lambda_{00}\rangle + 16\,|0100\rangle|\lambda_{01}\rangle + 16\,|1011\rangle|\lambda_{10}\rangle + 16\,|1011\rangle|\lambda_{11}\rangle\right]$$

4 qubit 2 qubit

$2^6 \times 1$

$$2^4 \begin{cases} P_{0100} = |0100\rangle\langle 0100| \otimes \mathbb{1}^{\otimes 2} \\ \quad (2^4 \times 2^4) \quad (2^2 \times 2^2) \\ \vdots \quad 2^6 \times 2^6 \\ P_{1011} = |1011\rangle\langle 1011| \otimes \mathbb{1}^{\otimes 2} \end{cases}$$

$\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$\downarrow$

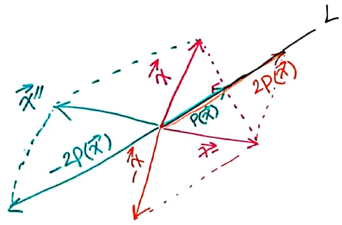
$Pr(\lambda = 0100)$

or

$Pr(\lambda = 1011)$

HW12. Q1.

reflection about a line L $\longrightarrow$ The formula is: $2P(\vec{x}) - \vec{x} = (2P - \mathbb{1})\vec{x} = \underline{\underline{\vec{x}'}}$



$(\mathbb{1} - 2P)\vec{x} = \vec{x}''$

$(-2P_1 + 1)(x|succ\rangle + y|fail\rangle)$

$-2P_1(x|succ\rangle + y|fail\rangle) + (x|succ\rangle + y|fail\rangle)$

$-2x|succ\rangle + (x|succ\rangle + y|fail\rangle)$

$-x|succ\rangle + y|fail\rangle$

Part (a): P_1 is the projective measurement for success, $\{x|succ\rangle + y|fail\rangle \mid x, y \in \mathbb{R}\}$

Prove that $\mathbb{1} - 2P_1$ is the reflection through $|fail\rangle$

Q2. part (a) $(\mathbb{1} - 2P_0)(x|succ\rangle + y|fail\rangle) = x|succ\rangle + y|fail\rangle - 2x|fail\rangle\cancelto{0}{\langle fail|succ\rangle} - 2y|fail\rangle\cancelto{1}{\langle fail|fail\rangle}$

$= x|succ\rangle - y|fail\rangle$

HW12.Q

* Grover search algorithm: ① $q[n], r[1] \longrightarrow |0^n\rangle \otimes |0\rangle = |0^m\rangle \to m = n+1$

② $\longrightarrow (H^{\otimes n} \otimes \mathbb{1})(|0^n\rangle \otimes |0\rangle) = H^{\otimes n}|0^n\rangle \otimes |0\rangle = |+^{\otimes n}\rangle |0\rangle \to \boxed{\text{first guess}}$

③ $\longrightarrow U_f(|+\rangle^{\otimes n}|0\rangle) = |+\rangle^{\otimes n} |f(|+\rangle^{\otimes n})\rangle$

$U_f = U_f^\dagger$

④ $\longrightarrow$ measure the $|r\rangle$ register

$U_f |x\rangle \otimes |y\rangle = |x\rangle \otimes |y \oplus f(x)\rangle$

$\xrightarrow{U_f} |x\rangle \otimes |y \oplus f(x) \oplus f(x)\rangle = |x\rangle \otimes |y\rangle$ $\quad U_f U_f = \mathbb{1} \Rightarrow U_f = U_f^\dagger \quad U_f = U_f^\dagger$

Q2. (part b) $A = G(2|0^m\rangle\langle 0^m| - \mathbb{1})G^\dagger$, $G := U_f(H^{\otimes n} \otimes \mathbb{1})$, $|g_1\rangle = G|0^m\rangle = U_f(H^{\otimes n} \otimes \mathbb{1})|0^m\rangle$

$= U_f(|+^{\otimes n}\rangle \otimes |0\rangle)$

reflection through $|g_1\rangle = 2|g_1\rangle\langle g_1| - \mathbb{1} = 2U_f(|+\rangle^{\otimes n}|0\rangle)(\langle +^{\otimes n}|\langle 0|)U_f - \mathbb{1}$ Ⓘ

$G(2|0^m\rangle\langle 0^m| - \mathbb{1})G^\dagger = U_f(H^{\otimes n} \otimes \mathbb{1})(2\underbrace{|0^m\rangle\langle 0^m|}_{|0^n\rangle\langle 0^n| \otimes |0\rangle\langle 0|} - \mathbb{1})(H^{\otimes n} \otimes \mathbb{1})U_f$

$= U_f(2|+\rangle^{\otimes n}\langle 0^n| \otimes |0\rangle\langle 0| - H^{\otimes n} \otimes \mathbb{1})(H^{\otimes n} \otimes \mathbb{1})U_f$

$= U_f(2|+\rangle^{\otimes n}\langle +|^{\otimes n} \otimes |0\rangle\langle 0| - \mathbb{1}^{\otimes m})U_f = 2U_f(|+\rangle^{\otimes n}|0\rangle)(\langle +|^{\otimes n}\langle 0|) - \underbrace{U_f U_f}_{\mathbb{1}} =$ Ⓘ

$\cdot\square$