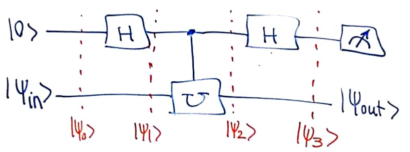
HW10. Q1:



$eig(U) \in \{\pm 1\}$

$$\begin{cases} \lambda_0 = (-1)^0 = 1, \ |\lambda_0\rangle \\ \lambda_1 = (-1)^1 = -1, \ |\lambda_1\rangle \end{cases}$$

$$|\psi_{in}\rangle = \alpha_0 |\lambda_0\rangle + \alpha_1 |\lambda_1\rangle \longrightarrow |\psi_0\rangle = |0\,\psi_{in}\rangle = \alpha_0 |0\lambda_0\rangle + \alpha_1 |0\lambda_1\rangle$$

$$|\psi_1\rangle = (H \otimes \mathbb{1})(|\psi_0\rangle) = \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right)\left(\alpha_0 |0\lambda_0\rangle + \alpha_1 |0\lambda_1\rangle\right) = \frac{\alpha_0 |0\lambda_0\rangle + \alpha_1 |0\lambda_1\rangle + \alpha_0 |1\lambda_0\rangle + \alpha_1 |1\lambda_1\rangle}{\sqrt{2}}$$

$(-1)^0 |\lambda_0\rangle = \lambda_0 |\lambda_0\rangle$ $\qquad$ $(-1)^1 |\lambda_1\rangle = \lambda_1 |\lambda_1\rangle$

$$|\psi_2\rangle = C\text{-}U(|\psi_1\rangle) = \frac{\alpha_0 |0\lambda_0\rangle + \alpha_1 |0\lambda_1\rangle + \alpha_0 |1\rangle U|\lambda_0\rangle + \alpha_1 |1\rangle U|\lambda_1\rangle}{\sqrt{2}} = \alpha_0 \left(\frac{|0\rangle + (-1)^0 |1\rangle}{\sqrt{2}}\right)|\lambda_0\rangle + \alpha_1 \left(\frac{|0\rangle + (-1)^1 |1\rangle}{\sqrt{2}}\right)|\lambda_1\rangle$$

$$|\psi_3\rangle = (H \otimes \mathbb{1})(|\psi_2\rangle) = \alpha_0 \, H\left(\frac{|0\rangle + (-1)^0 |1\rangle}{\sqrt{2}}\right)|\lambda_0\rangle + \alpha_1 \, H\left(\frac{|0\rangle + (-1)^1 |1\rangle}{\sqrt{2}}\right)|\lambda_1\rangle$$

$$H|x\rangle = \frac{|0\rangle + (-1)^x |1\rangle}{\sqrt{2}} \xrightarrow{\times H} |x\rangle = H\left(\frac{|0\rangle + (-1)^x |1\rangle}{\sqrt{2}}\right)$$

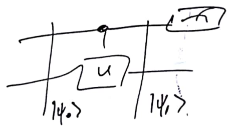
$$\longrightarrow |\psi_3\rangle = \alpha_0 |\underset{\downarrow \lambda_0}{0}\lambda_0\rangle + \alpha_1 |\underset{\downarrow \lambda_1}{1}\lambda_1\rangle \xrightarrow[\text{qubit}]{\text{Measure the first}} \begin{cases} Pr(0) = |\alpha_0|^2 : & |\psi_{out}\rangle = |\lambda_0\rangle \\ Pr(1) = |\alpha_1|^2 : & |\psi_{out}\rangle = |\lambda_1\rangle \end{cases}$$

HW10. Q2:

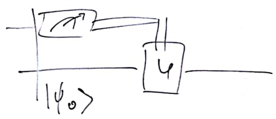
$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ - target

$|c\rangle = c_0|0\rangle + c_1|1\rangle$ - controlled

I



II



$|\psi_0\rangle = |c\psi\rangle = \alpha c_0|00\rangle + \beta c_0|01\rangle + \alpha c_1|10\rangle + \beta c_1|11\rangle$

$\xrightarrow{c\text{-}U} |\psi_1\rangle = \alpha c_0|00\rangle + \beta c_0|01\rangle + \alpha c_1|1\rangle U|0\rangle + \beta c_1|1\rangle U|1\rangle$

I

$P(q_1=0) = |\alpha c_0|^2 + |\beta c_0|^2 = |c_0|^2 \longrightarrow$ post-meas-state: $\alpha c_0|00\rangle + \beta c_0|01\rangle$

$P(q_1=1) = |\alpha c_1|^2 + |\beta c_1|^2 = |c_1|^2 \longrightarrow$ " : $\alpha c_1|1\rangle U|0\rangle + \beta c_1|1\rangle U|1\rangle$

|| equal

II

$P(q_1=0) = |\alpha c_0|^2 + |\beta c_0|^2 = |c_0|^2 \longrightarrow$ post-meas-state: $\alpha c_0|00\rangle + \beta c_0|01\rangle$

$P(q_1=1) = |\alpha c_1|^2 + |\beta c_1|^2 = |c_1|^2 \longrightarrow$: $\alpha c_1|1\rangle U|0\rangle + \beta c_1|1\rangle U|1\rangle$

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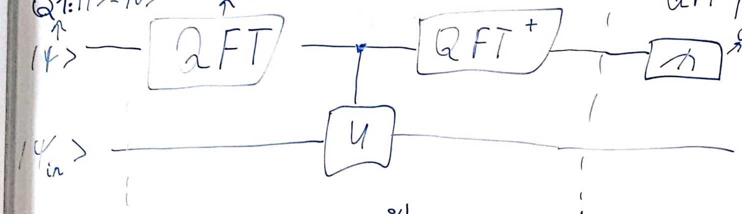
HW10. Q3: $\mathcal{U}$ $\quad$ $eig(\mathcal{U}) \in \{\pm 1\}$

QPE

Q1: $|\psi\rangle = |0\rangle$

Q1: $QFT|0\rangle = H|0\rangle$

$QFT|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$



if $|\psi\rangle = |0\rangle \longrightarrow$ Then this circuit is as in Exercise 1:

$QFT(|0\rangle) = H|0\rangle$

$$QFT^{+}\left(QFT(|x\rangle) = \frac{|0\rangle + e^{2\pi i [0/x]}|1\rangle}{\sqrt{2}} = |+\rangle \right.$$

0/1, 0/0

$H|0\rangle = |+\rangle$

$$\begin{cases} 0/x = 0/0 : 0 \\ 0/x = 0/1 : 2^{-1} = \frac{1}{2} \end{cases}$$

$$\frac{|0\rangle + (-1)^{1 \in [0,1]}|1\rangle}{\sqrt{2}}$$

HW9. Q3:

$$H = \frac{1}{2}\mathbb{1} + \frac{3}{16} X \otimes Y, \quad m=4, \quad |I\rangle := |11\rangle \xrightarrow{QPE} |R, I\rangle$$

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$X : |+\rangle, |-\rangle$

$Y : |\circlearrowright\rangle, |\circlearrowleft\rangle$

$\longrightarrow X \otimes Y :$

$$\begin{array}{cc} \frac{11}{16} & \frac{5}{16} \\ \uparrow & \uparrow \\ |+\circlearrowleft\rangle, & |+\circlearrowright\rangle \\ |-\circlearrowleft\rangle, & |-\circlearrowright\rangle \\ \downarrow & \downarrow \\ \frac{5}{16} & \frac{11}{16} \end{array}$$

$$H|+\circlearrowleft\rangle = \frac{1}{2}|+\circlearrowleft\rangle + \frac{3}{16} \underbrace{\underbrace{X|+\rangle}_{|+\rangle} \otimes \underbrace{Y|\circlearrowleft\rangle}_{|\circlearrowleft\rangle}}_{|+\circlearrowleft\rangle}$$

$$H|+\circlearrowleft\rangle = \left(\frac{1}{2} + \frac{3}{16}\right)|+\circlearrowleft\rangle = \frac{11}{16}|+\circlearrowleft\rangle$$

$$\lambda_1 = \frac{5}{16} \qquad |\lambda_1\rangle = \frac{|10\rangle + |11\rangle}{\sqrt{2}}$$

$$\lambda_2 = \frac{11}{16} \qquad |\lambda_2\rangle = \frac{-|10\rangle + |11\rangle}{\sqrt{2}}$$

$$H|\lambda_1\rangle = \frac{|10\rangle + |11\rangle}{2\sqrt{2}} + \frac{3}{16}\left(\quad |01\rangle \right.$$

$$\frac{1}{2} I + \frac{3}{16} X \otimes Y$$

$+1 \ \lambda_{x1} \quad +1 \ \lambda_{y1}$

$\frac{1}{2} \qquad -1 \ \lambda_{x2} \quad -1 \ \lambda_{y2}$

$$\frac{1}{2} + \frac{3}{16} \ 1 \cdot 1$$

$$\frac{1}{2} + \frac{3}{16} \ (-1) \cdot 1$$

$(-1)\ (-1)$

$1 \ (-1)$