$N := 2^n$

HW2. Q1) Prove $Z = \underbrace{|0\rangle\langle 0| - |1\rangle\langle 1|}_{A}$, $X = |+\rangle\langle +| - |-\rangle\langle -|$, $Y = |\circlearrowright\rangle\langle\circlearrowright| - |\circlearrowleft\rangle\langle\circlearrowleft|$

Sol 1.

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = Z$$

$$\begin{array}{cc} |0\rangle & |1\rangle \end{array}$$
$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} - \begin{pmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = X$$

$$\begin{pmatrix} 1/2 & -i/2 \\ i/2 & 1/2 \end{pmatrix} - \begin{pmatrix} 1/2 & i/2 \\ -i/2 & 1/2 \end{pmatrix} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = Y$$

Sol. If their action on the basis of Hilbert space were the same, then they are equal.

$B_Z = \{|0\rangle, |1\rangle\} \longrightarrow \begin{cases} Z|0\rangle = |0\rangle \\ Z|1\rangle = -|1\rangle \end{cases}, \begin{cases} A|0\rangle = |0\rangle \\ A|1\rangle = -|1\rangle \end{cases} \longrightarrow A = Z. \square$  $B_X = \{|+\rangle, |-\rangle\}$, $B_Y = \{|\circlearrowright\rangle, |\circlearrowleft\rangle\}$

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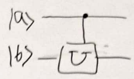
$N := 2^n$

HW2. Q2) $CU(|0\rangle|\varphi\rangle) = |0\rangle|\varphi\rangle$, $CU(|1\rangle|\varphi\rangle) = |1\rangle U|\varphi\rangle$




n-qubit

(a) $CU = |0\rangle\langle 0|\otimes 1 + |1\rangle\langle 1|\otimes U = A$   Sol 1.

(b) CU is unitary operator.

$|\psi\rangle = (\alpha|0\rangle + \beta|1\rangle)\otimes|\varphi\rangle =$

$\alpha|0\rangle_2|\varphi\rangle_n$ $2n$

$\beta|1\rangle|\varphi\rangle_{2n}$

$\alpha|0\rangle\otimes|\varphi\rangle + \beta|1\rangle\otimes|\varphi\rangle$

$\begin{pmatrix}1&0\\0&0\end{pmatrix}\otimes 1 + \begin{pmatrix}0&0\\0&1\end{pmatrix}\otimes U = \begin{pmatrix}1 & 0\\ 0 & U\end{pmatrix} =: A$

Sol2. (a)

- $A(|0\rangle|\varphi\rangle) = (|0\rangle\langle 0|\otimes 1 + |1\rangle\langle 1|\otimes U)(|0\rangle|\varphi\rangle)$

$= (|0\rangle\langle 0|0\rangle\otimes|\varphi\rangle) + (|1\rangle\langle 1|0\rangle\otimes U|\varphi\rangle)$

$= |0\rangle|\varphi\rangle = CU(|0\rangle|\varphi\rangle)$

- $A(|1\rangle|\varphi\rangle) = (|0\rangle\langle 0|1\rangle\otimes|\varphi\rangle) + (|1\rangle\langle 1|1\rangle\otimes U|\varphi\rangle)$

$= |1\rangle U|\varphi\rangle = CU(|1\rangle|\varphi\rangle)$

$CU = A$. □

$A(|\psi\rangle) = \begin{pmatrix}1&0\\0&U\end{pmatrix}|\psi\rangle = \alpha|0\rangle|\varphi\rangle + \beta|1\rangle U|\varphi\rangle$

$CU(|\psi\rangle) = A|\psi\rangle$   $(A\otimes B)^\dagger = A^\dagger\otimes B^\dagger$

(b) Unitary: $CU^\dagger = (|0\rangle\langle 0|\otimes 1 + |1\rangle\langle 1|\otimes U)^\dagger = |0\rangle\langle 0|\otimes 1 + |1\rangle\langle 1|\otimes U^\dagger$

$(CU^\dagger)(CU) = \begin{bmatrix}1&0\\0&U^\dagger\end{bmatrix}\begin{bmatrix}1&0\\0&U\end{bmatrix} = 1$.

$N := 2^n$

HW2. Q3. (a) ✓ last practice session

(b) $Z := |0\rangle\langle 0| - |1\rangle\langle 1|$ , $U := iZ \rightarrow$ quantum program which behaves differently

$U|\psi\rangle = iZ|\psi\rangle \rightarrow$ physically indist. from $Z|\psi\rangle$

$cU|+\rangle|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle\otimes|+\rangle + |1\rangle\otimes U|+\rangle)$

$= \frac{1}{\sqrt{2}}(|0\rangle\otimes|+\rangle + i|1\rangle\otimes Z|+\rangle)$

$= \left(\frac{1}{\sqrt{2}}(|0\rangle\otimes|+\rangle + i|1\rangle\otimes|-\rangle)\right)$

$cZ(cU|+\rangle|+\rangle) = \frac{1}{\sqrt{2}}(|0\rangle\otimes|+\rangle + i|1\rangle\otimes Z|-\rangle)$

$= \frac{1}{\sqrt{2}}(|0\rangle\otimes|+\rangle + i|1\rangle\otimes|+\rangle)$

$= \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)\otimes|+\rangle$

$= |\circlearrowleft\rangle|+\rangle$

$cZ|+\rangle|+\rangle = \frac{1}{\sqrt{2}}cZ(|0\rangle+|1\rangle)\otimes|+\rangle$

$= \frac{1}{\sqrt{2}}cZ(|0\rangle\otimes|+\rangle + |1\rangle\otimes|+\rangle)$

$= \frac{1}{\sqrt{2}}(|0\rangle\otimes|+\rangle + |1\rangle\otimes Z|+\rangle)$

$= \frac{1}{\sqrt{2}}(|0\rangle\otimes|+\rangle + |1\rangle\otimes|-\rangle)$

$= \frac{1}{\sqrt{2}}(|0\rangle\otimes|+\rangle$

$cZcZ|+\rangle|+\rangle = \frac{1}{\sqrt{2}}cZ$

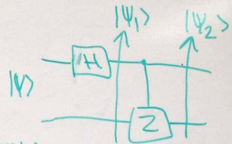
$= \frac{1}{\sqrt{2}}(|0\rangle\otimes|+\rangle + |1\rangle\otimes Z|-\rangle)$

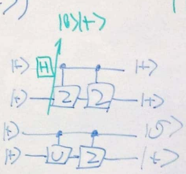
$= \frac{1}{\sqrt{2}}(|0\rangle\otimes|+\rangle + |1\rangle\otimes|+\rangle) = |+\rangle|+\rangle$

$Z|+\rangle = |-\rangle$ $\quad \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$Z|-\rangle = |+\rangle$

$|+\rangle = \frac{|0\rangle+|1\rangle}{\sqrt{2}} \xrightarrow{Z|+\rangle} \frac{|0\rangle-|1\rangle}{\sqrt{2}} = |-\rangle$




$|\psi_1\rangle = (H\otimes I)|\psi\rangle$

$|\psi_2\rangle = cZ|\psi_1\rangle$

$H|+\rangle = |0\rangle$

$cZ(|0\rangle|+\rangle)$

$(H\otimes I)(|+\rangle|+\rangle)$

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