HW1.

Q1. $|\psi\rangle,\ |\xi|=1 \longrightarrow |\psi\rangle,\ \xi|\psi\rangle$

$$\begin{cases} U(|\psi\rangle) = \overbrace{U|\psi\rangle}^{|\varphi\rangle} \\ U(\xi|\psi\rangle) = \xi U|\psi\rangle = \xi|\varphi\rangle \end{cases}$$

$P_A := |A\rangle\langle A|$

$P_B := |B\rangle\langle B|$

$\longrightarrow$ post_mea.-st. $= \dfrac{P_i|\psi\rangle}{\|P_i|\psi\rangle\|}$

$i \in \{A, B\}$

$$\frac{P_A|\psi\rangle}{\|P_A|\psi\rangle\|} = \frac{|A\rangle\langle A|\psi\rangle}{\|\quad '' \quad\|}$$

$$|\psi\rangle = \alpha|A\rangle + \beta|B\rangle \longrightarrow P_A|\psi\rangle = |A\rangle\langle A|(\alpha|A\rangle + \beta|B\rangle)$$

$$= \alpha|A\rangle\langle A|A\rangle + \beta|A\rangle\cancel{\langle A|B\rangle}^{0}$$

$$= \alpha|A\rangle$$

$P_B|\psi\rangle = \cdots = \beta|B\rangle$

$\longrightarrow$ P-m-s. $|A\rangle$

$\xi|\psi\rangle \longrightarrow |\alpha|^2 = |\alpha\xi|^2 : |A\rangle$

$|\beta|^2 = |\beta\xi|^2 : |B\rangle$

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R $\ldots |V^{\perp}\rangle \}$

HW1. $|\varphi\rangle = |0\rangle$, $|\psi\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle$;

* first basis: $B_1 = \{|0\rangle, |1\rangle\}$

$\longrightarrow$ apply on $|\varphi\rangle \longrightarrow \begin{cases} Pr(\text{meas.} = 0) = 1 \\ Pr(\text{meas.} = 1) = 0 \end{cases}$

$\longrightarrow$ apply on $|\psi\rangle \longrightarrow \begin{cases} Pr(m. = 0) = \cos^2\theta \\ Pr(m. = 1) = \sin^2\theta \end{cases}$

if the final result of measurement is

- 1 $\longrightarrow$ we are sure that we were measuring $|\psi\rangle$
- 0 $\longrightarrow$ we were either measuring $|\varphi\rangle$ or $|\psi\rangle$

$\longrightarrow$ so if we get 1 as the final result, we can distinguish the initial states.

* second basis: $B_2 = \{\cos\theta|0\rangle + \sin\theta|1\rangle, \sin\theta|0\rangle - \cos\theta|1\rangle\} \longrightarrow$ if we get 1, we know by sure that we were measuring $|\varphi\rangle \ldots$