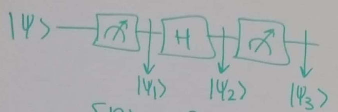
* A qubit is measured with respect to computational basis. After this a Hadamard transformation is applied to it. Then it is measured again with respect to comp. basis. what is the probability of observing '0' in the second measurement.

$B = \{|0\rangle, |1\rangle\}$

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$



$$|\psi_1\rangle = \begin{cases} |0\rangle & \alpha^2 \\ |1\rangle & \beta^2 \end{cases}$$

$$|\psi_2\rangle = H|\psi_1\rangle = \begin{cases} |+\rangle & \alpha^2 \\ |-\rangle & \beta^2 \end{cases}$$

$$|\psi_3\rangle = \begin{cases} |0\rangle & \frac{1}{2}\cdot\alpha^2 \\ |1\rangle & \frac{1}{2}\cdot\alpha^2 \\ |0\rangle & \frac{1}{2}\beta^2 \\ |1\rangle & \frac{1}{2}\beta^2 \end{cases} = \begin{cases} |0\rangle & \frac{\alpha^2+\beta^2}{2} = \frac{1}{2} \\ |1\rangle & \frac{\alpha^2+\beta^2}{2} = \frac{1}{2} \end{cases}$$

$$0: \frac{P_0|+\rangle}{\|P_0|+\rangle\|} = |0\rangle, \quad \|P_0|+\rangle\| = \frac{1}{\sqrt{2}}$$

$$1: \frac{P_1|+\rangle}{\|P_1|+\rangle\|} = |1\rangle, \quad \|P_1|+\rangle\| = \frac{1}{\sqrt{2}}$$

$\alpha^2 + \beta^2 = 1$

* Show that if we measure the first qubit of the Bell state $|\beta_{00}\rangle$ and observe '0', then after this the Probability of observing '0' when measuring the second qubit is 1.

$$|ab\rangle \neq |\beta_{00}\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}} \qquad P_0 = |0\rangle\langle 0|, \; P_1 = |1\rangle\langle 1|$$

$$|\beta_{01}\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}$$

$$|\beta_{10}\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}}$$

$$|\beta_{11}\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

$$B = \{|\beta_{00}\rangle, |\beta_{01}\rangle, |\beta_{10}\rangle, |\beta_{11}\rangle\}$$

$$\begin{cases} |00\rangle & \frac{1}{2} \\ |11\rangle & \frac{1}{2} \end{cases}$$