Operational Semantics
Natural Semantics
2.1 Natural Semantics

Specification

- Natural semantics of statements (blackboard):
  - Configurations and terminal configurations.
  - Transitions.
  - Axioms and rules (Table 2.1).
Examples and exercises

- Improvised example about assignment (blackboard).
- Example 2.1 and a symmetric example (blackboard).
- Example 2.2 (blackboard).
- Exercise 2.3 (oneself).
Termination and looping

• Definitions (blackboard).

• Exercise 2.4 (oneself)
Determinism

- Definition (blackboard).
- Theorem 2.9 (blackboard).
2 Operational Semantics

2.1 Natural Semantics

**Denotations**

- Semantic function (blackboard):
  - Type;
  - Definition;
  - Discussion on compositionality.
2 Operational Semantics
2.1 Natural Semantics

Semantic equivalence

- Exercise

\[ \text{skip} ; S \sim_{ns} S \sim_{ns} S \ ; \text{skip} \]

(blackboard).

- Exercise 2.6 (home).

- Lemma 2.5 (blackboard).

- Exercise 2.13 (blackboard).
2 Operational Semantics
2.1 Natural Semantics

Operational semantics of expressions

• Exercise 2.11 (blackboard):
  – Configurations and terminal configurations;
  – Transitions;
  – Axioms and rules.

• Exercise 2.12 (home):
  – Configurations and terminal configurations;
  – Transitions;
  – Axioms and rules.
2 Operational Semantics

2.2 Structural Operational Semantics

Structural Operational Semantics
Specification

- Structural semantics of statements (blackboard):
  - Configurations and terminal configurations;
  - Transitions;
  - Axioms and rules (Table 2.2).

- Derivation sequences (blackboard):
  - Definition;
  - Stuck configurations.
Examples and exercises

- Example 2.14 and a symmetric example (blackboard).
- Example 2.15 (blackboard).
- Exercise 2.16 (oneself).
Termination and looping

• Definitions (blackboard).

• Exercise 2.4 revisited in the context of structural semantics (oneself).
Important facts

• Lemma 2.19 (blackboard).
• Lemma 2.21 (blackboard).
• Exercise 2.20 (oneself).
Determinism

- Definition (blackboard).

- Exercise 2.22 (blackboard).
Denotations

- Semantic function (blackboard):
  - Type;
  - Definition;
  - Discussion on compositionality.
Semantic equivalence

• Definition (blackboard).

• Exercise 2.23 (blackboard).

• Redefining the semantics of while-loops equivalently without referring to non-constituent syntactic objects (home).

• Exercise 2.25 (blackboard).
Equivalence of Semantics
An equivalence result

• Natural and structural semantics specify the same semantic function in different ways (blackboard):
  – Lemma 2.27;
  – Lemma 2.28;
Extensions of While
Abortion

• Abstract syntax (blackboard).

• Natural semantics (blackboard).

• Structural semantics (blackboard).

• Semantic functions (blackboard):
  – Type;
  – Definition.
Examples and exercises

• An improvised example (blackboard).

• Discussion (blackboard):
  – Show:
    \[
    \begin{align*}
    \text{abort } & \not\sim_{\text{ns}} \text{ skip;} \\
    \text{abort } & \sim_{\text{ns}} \text{ while true do skip;} \\
    \text{abort } & \not\sim_{\text{sos}} \text{ skip;} \\
    \text{abort } & \not\sim_{\text{sos}} \text{ while true do skip;}
    \end{align*}
    \]
  – Looping and abnormal termination are not distinguished by natural semantics.

• Exercise 3.1 (blackboard).
Non-determinism

- Abstract syntax (blackboard).
- Natural semantics (blackboard).
- Structural semantics (blackboard).
- Semantic functions (blackboard):
  - Type;
  - Definition.
Examples and exercises

• Let $S = x := 1 \text{ or } (x := 2 ; x := x + 2)$. Find (blackboard):
  – All natural semantics derivation trees of $S$ for initial state $s = \{x \mapsto 0\}$;
  – All structural semantics derivation sequences of $S$ for initial state $s$.

• Discussion (blackboard):
  – Show:
    
    \[
    \text{while } \text{true} \text{ do skip or } x := 1 \sim_{\text{ns}} x := 1;
    \]
    \[
    \text{while } \text{true} \text{ do skip or } x := 1 \not\sim_{\text{sos}} x := 1;
    \]

  – Non-determinism suppresses looping in natural semantics.

• Exercise 3.3 (oneself).
2 Operational Semantics
2.4 Extensions of While

Parallelism

• Abstract syntax (blackboard).
• Natural semantics (blackboard).
• Structural semantics (blackboard).
• Semantic functions (blackboard):
  – Type;
  – Definition.
Examples and exercises

- Let $S = x := 1 \text{ par } (x := 2 ; x := x + 2)$. Find (blackboard):
  - All natural semantics derivation trees of $S$ for initial state $s = \{x \mapsto 0\}$;
  - All structural semantics derivation sequences of $S$ for initial state $s$.

- Discussion (blackboard):
  - Natural semantics is unable to express interleaving of computations.
Atomic computations

- Abstract syntax (blackboard).
- Structural semantics (blackboard).
- Find a complete derivation sequence of

\[ x := 1 \text{ par atomic } x := 2 \ ; \ x := x + 2 \ \text{end} \]

for initial state \( s = \{ x \mapsto 0 \} \) (home).