MTAT.03.286: Design and Analysis of Algorithms **University of Tartu**

Midterm exam

December 16th, 2019

Student name: ____

Student ID: ____

1. This exam contains 8 pages. Check that no pages are missing.
2. It is possible to collect up to 120 points. Try to collect as many points as possible.
3. Justify and prove all your answers (where applicable). Show all important steps in your solution.
4. All facts and results that were proved or stated in the class can be used in your solution without a proof. Such results need to be rigorously formulated.
5. Any printed and written material is allowed in the class. No electronic devices are allowed.
6. Exam duration is 1 hour 40 minutes.
7. Good luck!

Question 1	
Question 2	
Question 3	
Total	

Question 1 (40 points). Let $\mathcal{G}(\mathcal{V}, \mathcal{E})$ be an undirected finite graph with a positive weight function

$$w : \mathcal{E} \to \mathbb{R}^+ ,$$

and $\mathcal{T}$ be a minimum spanning tree of $\mathcal{G}$. We add a new edge $e_0 = \{u, v\}$ to $\mathcal{G}$, where $u, v \in \mathcal{V}$. Denote the resulting graph $\mathcal{G}'(\mathcal{V}, \mathcal{E}')$, where $\mathcal{E}' = \mathcal{E} \cup \{e_0\}$. It is known that $w(e_0) > w(e)$ for any $e \in \mathcal{E}$ (in other words, the new edge has weight larger than any of the previously existing edges in $\mathcal{G}$).

Prove that $\mathcal{T}$ is also a minimum spanning tree of $\mathcal{G}'$.

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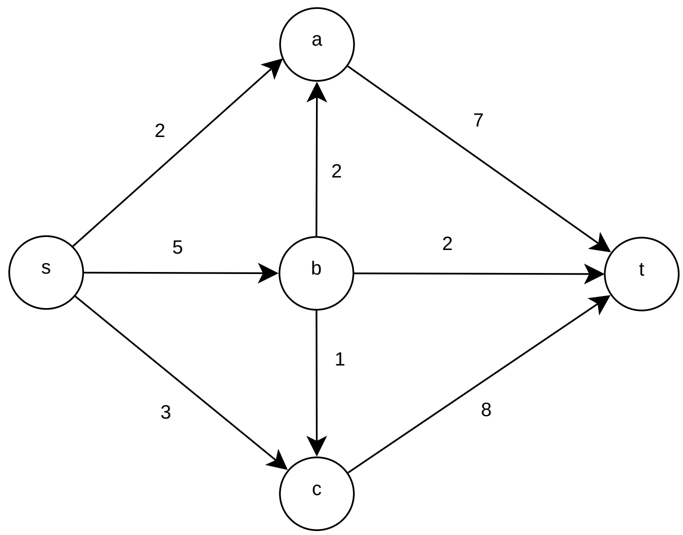
Question 2 (40 points). Find a maximum flow between s and t in the following network by using Dinitz algorithm:



Demonstrate the main steps in the algorithm. Show all minimum cuts.

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Question 3 (40 points). Let $\mathcal{N}(\mathcal{G}(\mathcal{V},\mathcal{E}),s,t,c)$ be a flow network. It is known that the maximum total flow from s to t in $\mathcal{N}$ is equal to F. Assume that the edge $e \in \mathcal{E}$ of capacity $c(e)$ belongs to a minimum cut between s and t.

Prove or disprove the following statements.

(a) If the value of $c(e)$ is decreased by $\Delta > 0$, then the maximum total flow from s to t in $\mathcal{N}$ becomes $F - \Delta$.

(b) If the value of $c(e)$ is increased by $\Delta > 0$, then the maximum total flow from s to t in $\mathcal{N}$ becomes $F + \Delta$.

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