Midterm exam

November 11th, 2019

Student name: ______________________________

Student ID: ______________________________

1. This exam contains 8 pages. Check that no pages are missing.
2. It is possible to collect up to 120 points. Try to collect as many points as possible.
3. Justify and prove all your answers (where applicable). Show all important steps in your solution.
4. All facts and results that were proved or stated in the class can be used in your solution without a proof. Such results need to be rigorously formulated.
5. Any printed and written material is allowed in the class. No electronic devices are allowed.
6. Exam duration is 1 hour 40 minutes.
7. Good luck!

Question 1	
Question 2	
Question 3	
Total	

Question 1 (40 points). Let $\mathcal{G}(\mathcal{V}, \mathcal{E})$ be a finite connected undirected graph without self-loops. Let $w : \mathcal{E} \mapsto \mathbb{R}$ be a weight function, such that $w(e) > 0$ for all edges $e \in \mathcal{E}$. Additionally, it is known that the weights of all edges in $\mathcal{G}$ are different.

Prove or disprove the following statements.

(a) If $\mathcal{T}(\mathcal{V}, \mathcal{E}_T)$ is a minimum spanning tree of $\mathcal{G}$, and e is the edge of the *minimum* weight in $\mathcal{G}$, then it holds that $e \in \mathcal{E}_T$.

(b) If $\mathcal{T}(\mathcal{V}, \mathcal{E}_T)$ is a minimum spanning tree of $\mathcal{G}$, and e is the edge of the *maximum* weight in $\mathcal{G}$, then it holds that $e \notin \mathcal{E}_T$.

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Question 2 (50 points). Let $\mathcal{N}(\mathcal{G}, s, t, c)$ be a flow network, where $\mathcal{G}(\mathcal{V}, \mathcal{E})$ is a finite directed graph. Prove that there exists a *maximum* flow function in $\mathcal{N}$ that does not contain a cycle.

Defintion: Let $f : \mathcal{E} \mapsto \mathbb{R}$ be a non-negative real-valued flow function in $\mathcal{N}$. We say that f *contains a cycle* if there exists a simple circuit in $\mathcal{G}$, such that for every edge e in this circuit $f(e) > 0$.

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Question 3 (30 points).

By using the Fast Fourier Transform (FFT) algorithm, evaluate the polynomial $P(x) = 2x^3 - x^2 - 4x + 6$ at the complex 4th roots of unity. Show at least one level of recursion.

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