Homework assignment 3

Due date: November 11, 2019

It is possible to collect up to 110 points in this homework assignment.

1. (a) Find a legal flow from $s$ to $t$ in the following network with upper and lower bounds. (You don’t have to specify all the steps in Ford-Flukerson or Dinitz algorithm that you are using, but you have to explain the construction and the resulting flow.)

(b) Find a maximum flow in the network in part (a). Show all minimum cuts.

2. By using the Fast Fourier Transform (FFT) algorithm, evaluate the polynomial $A(x) = x^4 + 2x^3 - x^2 + x + 1$ at the complex 6-th roots of unity. Show at least one level of recursion.

3. Let $A(x) = x^2 + 2x - 2$ and $B(x) = x + 2$. In this question, we will compute the polynomial $C(x) = A(x) \cdot B(x)$ by using the FFT algorithm.

(a) What is the minimum number of points we need to use? Explain.
(b) Evaluate $A(x)$ at the complex 4th roots of unity. Show at least one level of recursion.
(c) Evaluate $B(x)$ at the complex 4th roots of unity. Show at least one level of recursion.
(d) Compute $C(x)$ at the complex 4th roots of unity.
(e) Find the coefficients of $C(x)$.

4. Consider a family of matrices $M_0, M_1, M_2, \cdots$, which is defined recursively as follows:

- The matrix $M_0$ is $1 \times 1$ matrix
  \[ M_0 = \begin{pmatrix} 1 \end{pmatrix}. \]
- For $k > 0$, the $2^k \times 2^k$ matrix $M_k$ is defined by:
  \[ M_k = \begin{pmatrix} M_{k-1} & 3M_{k-1} \\ \frac{1}{2}M_{k-1} & \frac{3}{4}M_{k-1} \end{pmatrix}. \]
Show that if $\mathbf{v}$ is a column vector of length $n = 2^k$, then the product $H_k \mathbf{v}$ can be computed in time complexity $O(n \log n)$. Assume that addition of two numbers and multiplication of two numbers take constant time.