MTAT.03.286: Design and Analysis of Algorithms **University of Tartu**

Final exam

December 16th, 2019

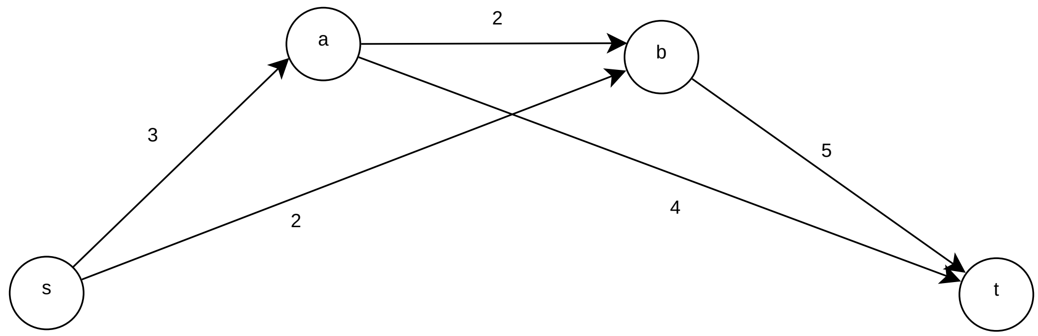
Student name: ____

Student ID: ____

1. This exam contains 8 pages. Check that no pages are missing.
2. It is possible to collect up to 120 points. Try to collect as many points as possible.
3. Justify and prove all your answers (where applicable).
4. All facts and results that were proved or stated in the class can be used in your solution without a proof. Such results need to be rigorously formulated.
5. Any printed and written material is allowed in the class. No electronic devices are allowed.
6. Exam duration is 1 hour and 40 minutes.
7. Good luck!

Question 1	
Question 2	
Question 3	
Total	

Question 1 (30 points). Consider the following flow network $\mathcal{N}(\mathcal{G}(\mathcal{V}, \mathcal{E}), s, t, c)$:



(a) Describe the problem of finding maximum flow from s to t in $\mathcal{N}$ as a linear programming problem, where variables $f_{sa}, f_{sb}, f_{ab}, f_{at}, f_{bt}$ denote the flow values in the corresponding edges.

(b) For each of the following sets of values, answer whether it corresponds to a feasible solution and whether it corresponds to an optimal feasible solution. Justify your answers.

(i) $f_{sa} = 3, f_{sb} = 2, f_{ab} = 1, f_{at} = 2, f_{bt} = 3$;

(ii) $f_{sa} = 2, f_{sb} = 2, f_{ab} = 0, f_{at} = 2, f_{bt} = 2$;

(iii) $f_{sa} = 3, f_{sb} = 2, f_{ab} = 2, f_{at} = 4, f_{bt} = 5$.

If you use an algorithm for finding a maximum flow, there is no need to show its execution steps.

Question 2 (50 points). You are given the following linear-programming (LP) problem:

$$\begin{array}{rrcrcrcr}
\textbf{max} & x_1 & - & 2x_2 & + & x_3 & & \\
\textbf{s.t.} & x_1 & + & x_2 & & & \leq & 3 \\
 & x_1 & & & + & x_3 & \leq & 5 \\
 & & - & x_2 & + & x_3 & \leq & 1 \\
 & \multicolumn{7}{l}{x_1 \geq 0, \;\; x_2 \geq 0, \;\; x_3 \geq 0}
\end{array}$$

(a) Solve this LP problem by using the simplex method.

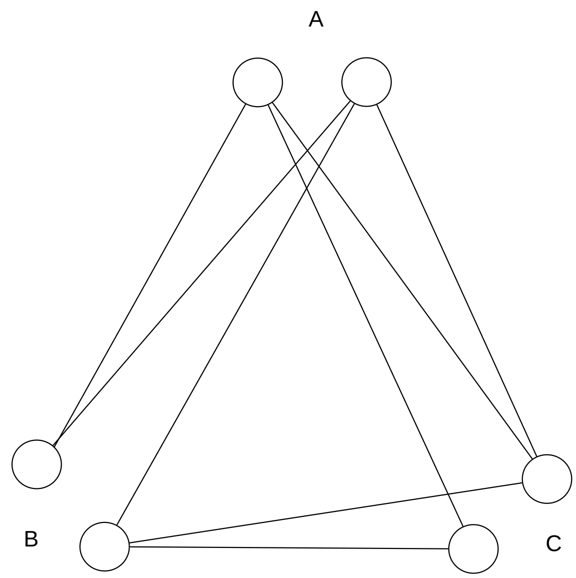
(b) Formulate the dual LP problem.

(c) Verify that your solution of (a) is optimal by the substitution of the point $(1, 0, 1)$ into the dual problem. Explain your answer.

Question 3 (40 points).

Definition: a *tripartite graph* is a graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$, whose vertex set is a union of three disjoint sets $\mathcal{V} = A \cup B \cup C$, such that there are no edges between any two vertices in the same set A, B or C.

Example:



Reminder: an *independent set* in a graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$ is a subset of vertices $\mathcal{S} \subseteq \mathcal{V}$ such that no two vertices in $\mathcal{S}$ are connected by an edge.

Assume that $\mathcal{G}(\mathcal{V}, \mathcal{E})$ is an undirected finite tripartite graph, $\mathcal{V} = A \cup B \cup C$. Propose a simple algorithm for finding a largest independent set with approximation factor of $\frac{1}{3}$. Prove correctness of your solution.