MTAT.03.286: Advanced Methods in Algorithms **University of Tartu**

Final exam

Instructor: Dr. Vitaly Skachek January 16th, 2014

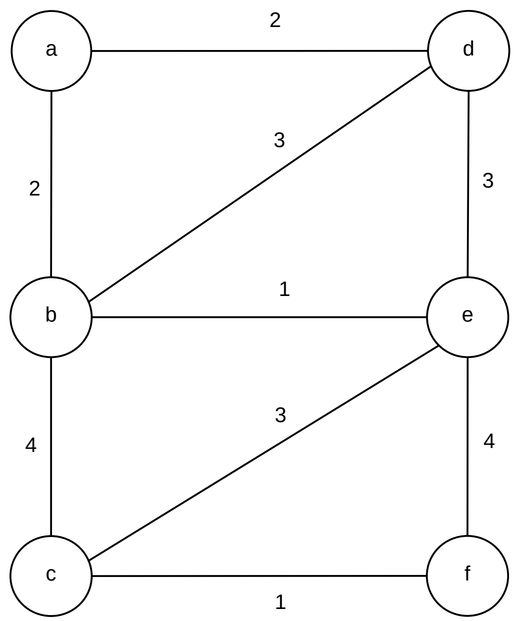
Student name: ______________________________

Student ID: ______________________________

1. This exam contains 10 pages. Check that no pages are missing.
2. It is possible to collect up to 120 points. Try to collect as many points as possible.
3. Justify and prove all your answers (where applicable).
4. All facts and results that were proved or stated in the class can be used in your solution without a proof. Such results need to be rigorously formulated.
5. Any printed and written material is allowed in the class. No electronic devices are allowed.
6. Exam duration is 2 hours.
7. Good luck!

Question 1	
Question 2	
Question 3	
Question 4	
Total	

Question 1 (20 points). Find a minimum spanning tree in the following graph by using one of the algorithms studied in the class. Show all the stages of the algorithm run.



Question 2 (30 points). You are given the following linear-programming (LP) problem:

$$
\begin{array}{llcl}
\textbf{max} & x_1 + x_2 + 2x_3 & & \\
\textbf{s.t.} & x_1 + x_2 + x_3 & \leq & 4 \\
& \phantom{x_1 + {}} 3x_2 + x_3 & \leq & 5 \\
& 2x_1 \phantom{{}+ x_2} - x_3 & \geq & 0 \\
& x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0 & &
\end{array}
$$

(a) Solve this LP problem by using the simplex method.

(b) Formulate the dual LP problem.

Question 3 (40 points). Let $\mathcal{N}(\mathcal{G}(\mathcal{V}, \mathcal{E}), s, t, c)$ be a flow network, where s is a source, t is a sink, $c : \mathcal{E} \to \mathbb{N}^+$ is a positive integer capacity function.

(a) Propose an efficient algorithm that, given an edge $e \in \mathcal{E}$, $c(e) > 0$, decides whether e belongs to **all** minimum cuts between s and t in $\mathcal{G}$.

(b) Propose an eficient algorithm that, given an edge $e \in \mathcal{E}$, $c(e) > 0$, decides whether e belongs to **some** minimum cut between s and t in $\mathcal{G}$.

In both parts of the question, prove the correctness of your solution and analyze its complexity.

Question 4 (30 points).

Let $\mathcal{G}(\mathcal{V}, \mathcal{E})$ be a finite undirected **bipartite** graph, $\mathcal{V} = \mathcal{A} \cup \mathcal{B}$, $\mathcal{A} \cap \mathcal{B} = \varnothing$, and all edges in the graph have one endpoint in $\mathcal{A}$ and one endpoint in $\mathcal{B}$. Such a graph will be called **thin** if the degree of every vertex in $\mathcal{A}$ is exactly 2. A subset of vertices $\mathcal{S} \subseteq \mathcal{B}$ is called **a set of representatives** if every vertex $v \in \mathcal{A}$ has at least one neighbor in $\mathcal{S}$.

Give an efficient factor 2 approximation algorithm that finds the smallest set of representatives in the thin graph as above. Proof its correctness and approximation factor.