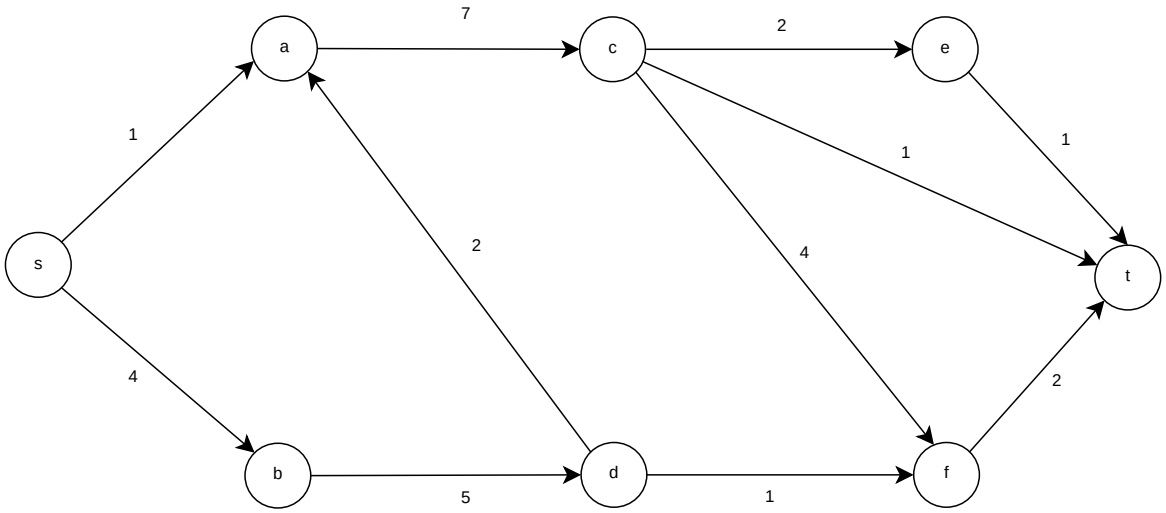
MTAT.03.286: Advanced Methods in Algorithms **University of Tartu**

Homework 2

Due date: October 24, 2014

It is possible to collect up to 110 points in this homework.

1. Find a maximum flow between s and t in the following network using Dinitz algorithm:



   Show the minimum cut. How many different minimum cuts can you find?

2. Let $\mathcal{H}(\mathcal{G}(\mathcal{V}, \mathcal{E}), s, t, c)$ be a network, where $\mathcal{G}$ is a finite directed graph, and $c : \mathcal{E} \to \mathbb{R}^+$ is a capacity function. For any two vertices $x, y \in \mathcal{E}$, denote by $F_{x,y}$ the maximum flow in $\mathcal{N}$ when x is the source and y is the sink. Prove that for all vertices $u, v, w \in \mathcal{E}$,

$$F_{u,w} \geq \min\{F_{u,v}, F_{v,w}\} .$$

3. Let $\mathcal{N}(\mathcal{G}(\mathcal{V}, \mathcal{E}), s, t, c)$ be a flow network, where s is a source, t is a sink, $c : \mathcal{E} \to \mathbb{Q}^+$ is a positive rational capacity function.

   (a) Propose an efficient algorithm that, given an edge $e \in \mathcal{E}$, $c(e) > 0$, decides whether e belongs to **all** minimum cuts between s and t in $\mathcal{G}$.

   (b) Propose an eficient algorithm that, given an edge $e \in \mathcal{E}$, $c(e) > 0$, decides whether e belongs to **some** minimum cut between s and t in $\mathcal{G}$.

   In both parts of the question, prove the correctness of your solution and analyze its complexity.

4. In the Institute of Computer Science, n students work as supervisors at a computer lab. During the week, there are m different supervision time slots. All the students have submitted the lists of the slots they can work at.

Propose an algorithm that assigns the time slots to the students, under the following conditions:

- Each student will obtain only slots, which he included in his list.
- At each time slot, there will be exactly two supervisors in the lab.
- No student will be scheduled for more than five slots on the same week.

If there is no legal arrangement of the slots to the students, the algorithm will output an appropriate message. The required time complexity is $O(nm^2)$.