MTAT.03.286: Advanced Methods in Algorithms **University of Tartu**

Homework 1

Due date: October 1, 2014

It is possible to collect up to 110 points in this homework.

1. For each of the following, indicate whether $f(n) = O(g(n))$, $f(n) = \Omega(g(n))$, or $f(n) = \Theta(g(n))$. Justify your answer.
   - $f(n) = 3^n$ and $g(n) = 2^n \cdot n^2$;
   - $f(n) = n^{\log_2 n}$ and $g(n) = 2^{\sqrt{n}}$;
   - $f(n) = n^{10}$ and $g(n) = (\log n)^{\log_2 n}$.

2. Let $T_1(V, E_1)$ and $T_2(V, E_2)$ be two spanning trees of the (undirected, connected, finite) graph $G(V, E)$. Prove that for every edge $e \in E_1 \backslash E_2$ there exists an edge $e' \in E_2 \backslash E_1$, such that *each* of the edge sets
$$(E_1 \cup \{e'\}) \backslash \{e\} \quad \text{and} \quad (E_2 \cup \{e\}) \backslash \{e'\}$$
defines a spanning tree.

3. Let $G(V, E)$ be an undirected, connected, finite graph with weight function $w : E \to \mathbb{R}^+$. Let T be a minimum spanning tree of G. Show that there exists a choice of the vertex s and of edges in Prim's algorithm that results in output T.

4. Let $G(V, E)$ be an undirected, connected, finite graph. Let $w : E \to \mathbb{R}^+$ and $w' : E \to \mathbb{R}^+$ be two weight functions, such that for all $e_1, e_2 \in E$:
$$w(e_1) \leq w(e_2) \iff w'(e_1) \leq w'(e_2) \ .$$
Prove that T is a minimum spanning tree of G with respect to w if and only if T is a minimum spanning tree of G with respect to w'.