MTAT.03.286: Advanced Methods in Algorithms **University of Tartu**

Homework 2

Due date: October 17, 2013

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1. Let $T_1(V, E_1)$ and $T_2(V, E_2)$ be two spanning trees of the (undirected, connected, finite) graph $G(V, E)$. Prove that for every edge $e \in E_1 \backslash E_2$ there exists an edge $e' \in E_2 \backslash E_1$, such that each of the edge sets
$$(E_1 \cup \{e'\}) \backslash \{e\} \quad \text{and} \quad (E_2 \cup \{e\}) \backslash \{e'\}$$
defines a spanning tree.

2. Let $G(V, E)$ be an undirected, connected, finite graph with weight function $w : E \to \mathbb{R}^+$. Let T be a minimum spanning tree of G. Show that there exists a run of Kruskal's algorithm that finds T (for suitable ordering of edges).

3. Let $G(V, E)$ be an undirected, connected, finite graph with weight function $w : E \to \mathbb{R}^+$. It is known that the weights of the edges in E are all different. Show that G has a *unique* minimum spanning tree.

4. Let $G(V, E)$ be an undirected, connected, finite graph. Let $w : E \to \mathbb{R}^+$ and $w' : E \to \mathbb{R}^+$ be two weight functions, such that for all $e_1, e_2 \in E$:
$$w(e_1) \leq w(e_2) \quad \Longleftrightarrow \quad w'(e_1) \leq w'(e_2)\ .$$
Prove that T is a minimum spanning tree of G with respect to w if and only if T is a minimum spanning tree of G with respect to w'.