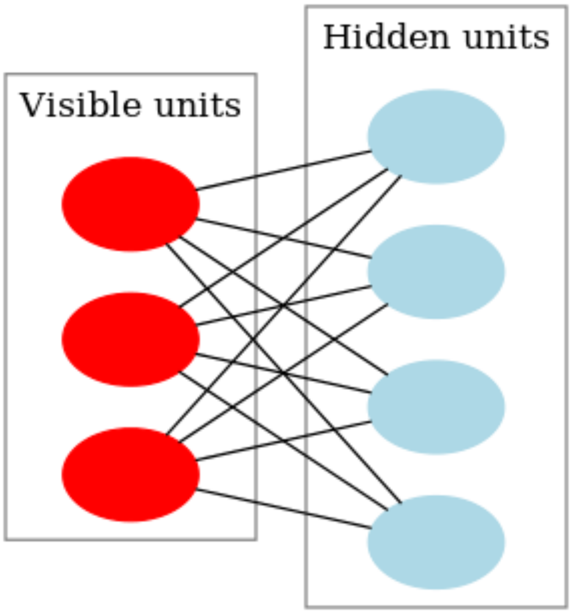
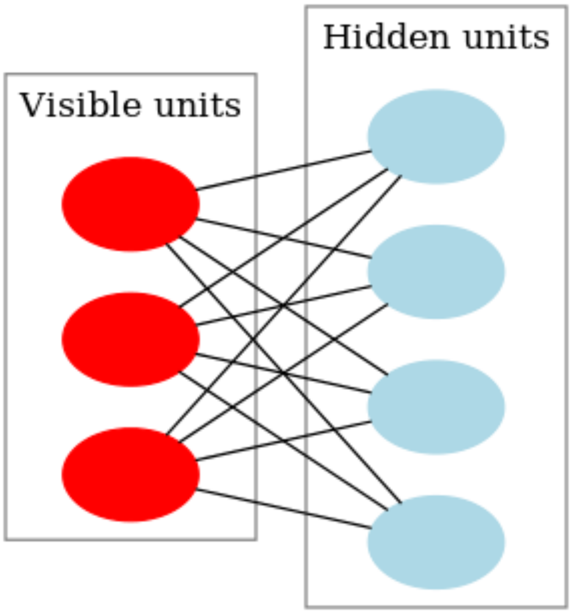
What happens when we stack RBMs?

Restricted Boltzmann Machine (RBM)



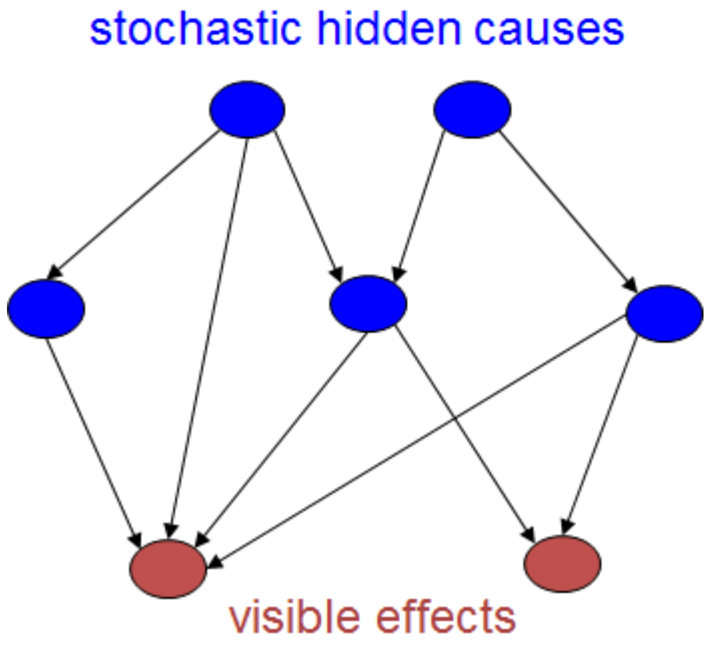
Restricted Boltzmann Machine (RBM)

★ Models probability distribution of inputs.
★ Can be thought of as dimensionality reduction.
★ Learns to generate pictures of cats.
★ Easy to train.

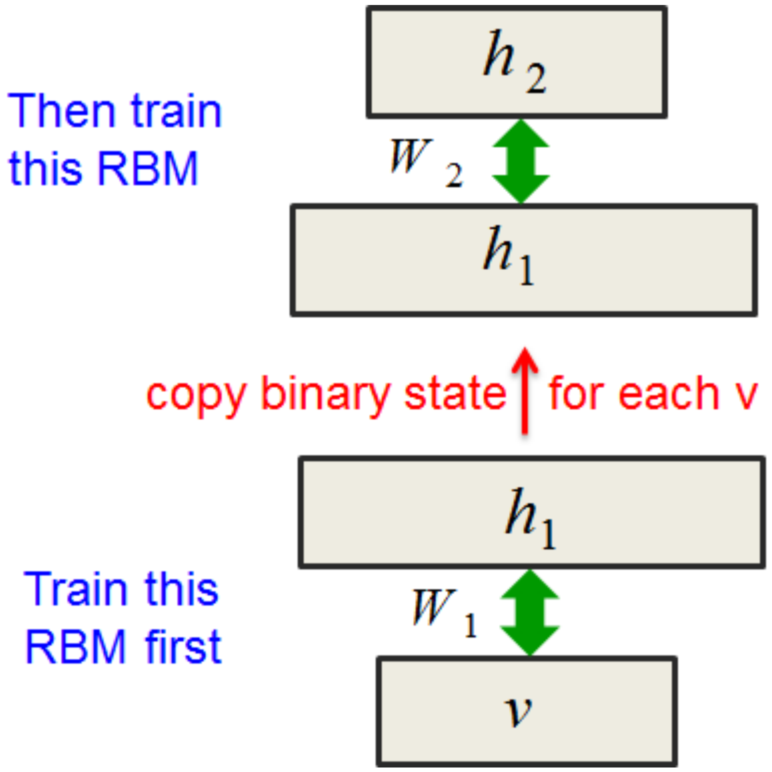


Sigmoid Belief Nets

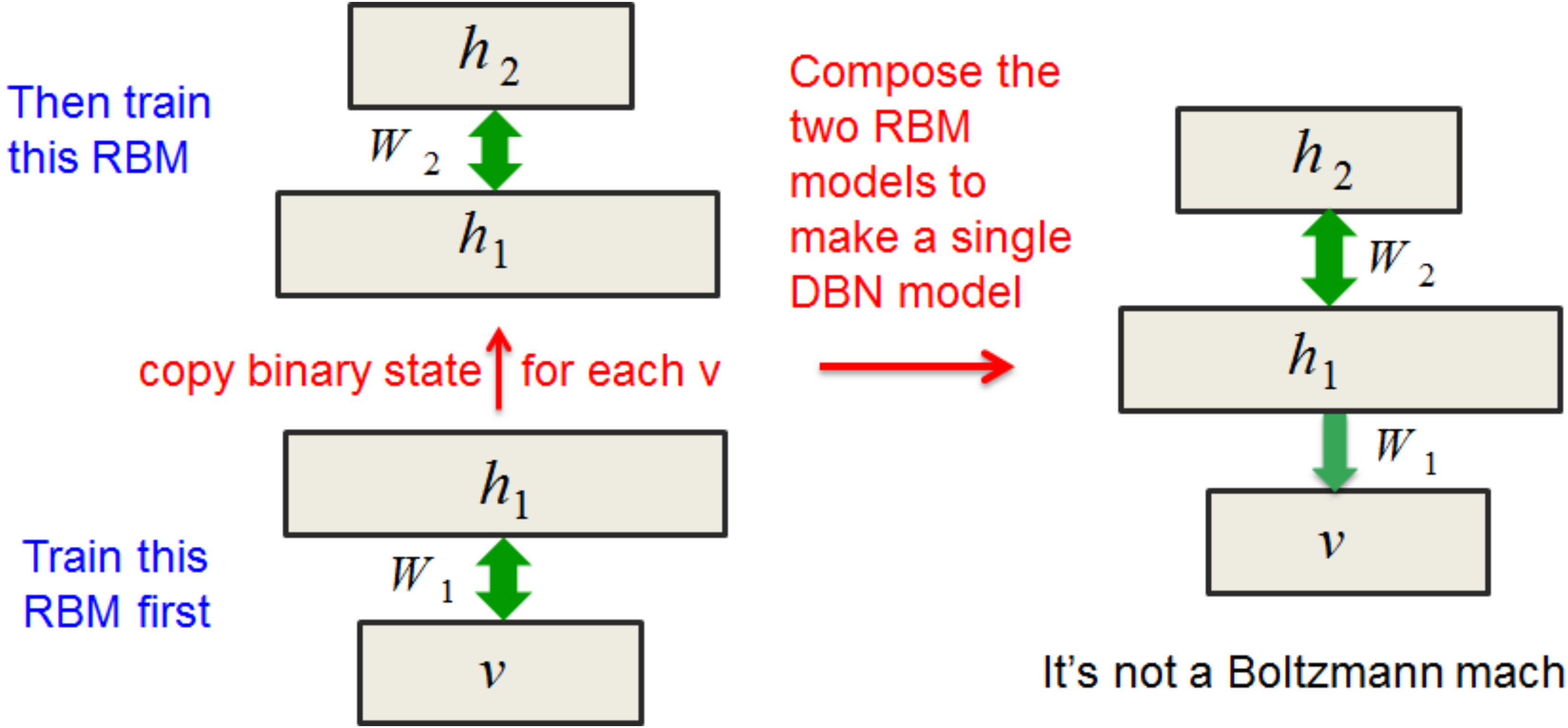
- ★ Would be better at generating pictures of cats (more layers).
- ★ Problems with training.



Stacking RBMs



Stacking RBMs

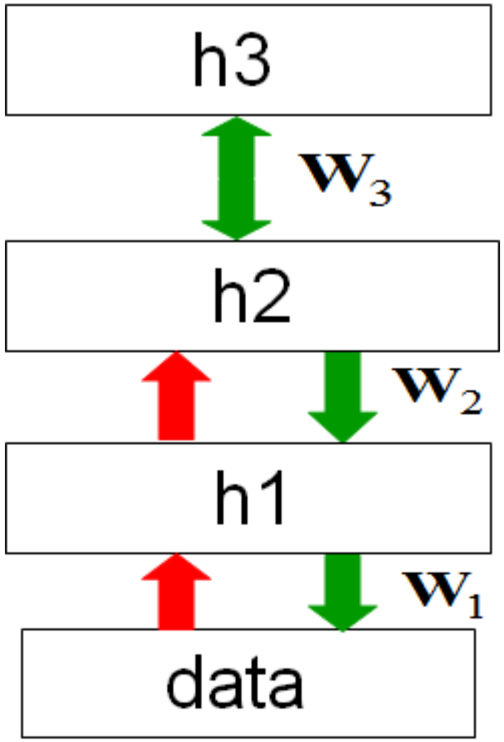


It's not a Boltzmann machine!

Generative model

To generate data:

1. Get an equilibrium sample from the top-level RBM by performing alternating Gibbs sampling for a long time.
2. Perform a top-down pass to get states for all the other layers.



Why does greedy learning work?

The weights, W, in the bottom level RBM define many different distributions: p(v|h); p(h|v); p(v,h); p(h); p(v).

We can express the RBM model as $$p(v) = \sum_h p(h)\ p(v \mid h)$$

If we leave p(v|h) alone and improve p(h), we will improve p(v).

To improve p(h), we need it to be a better model than p(h;W) of the aggregated posterior distribution over hidden vectors produced by applying W transpose to the data.

Fine-tuning

After learning many layers of features, we can fine-tune the features to improve generation.

1. **Do a stochastic bottom-up pass**

Then adjust the top-down weights of lower layers to be good at reconstructing the feature activities in the layer below.

2. **Do a few iterations of sampling in the top level RBM**

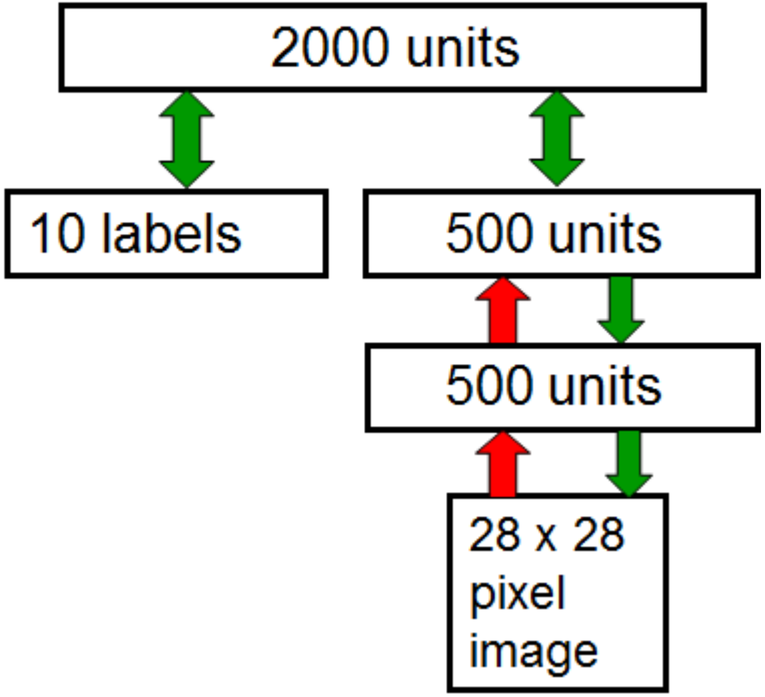
Then adjust the weights in the top-level RBM using CD.

3. **Do a stochastic top-down pass**

Then adjust the bottom-up weights to be good at reconstructing the feature activities in the layer above.

The DBN used for modeling the joint distribution of MNIST digits and their labels

- ★ The first two hidden layers are learned without using labels.
- ★ The top layer is learned as an RBM for modeling the labels concatenated with the features in the second hidden layer.
- ★ The weights are then fine-tuned to be a better generative model using contrastive wake-sleep.



Discriminative fine-tuning for DBNs

Fine-tuning for generation vs discrimination

- ★ Learn one layer at the time by stacking RBM-s
- ★ Treat as pre-training for initial set of weights
- ★ Generation - Contrastive wake-sleep
- ★ Discrimination - Backpropagation
  - ○ Can overcome many of the limitations of standard backpropagation
  - ○ Easier to learn deep nets
  - ○ Nets generalize better

Why backpropagation works better with greedy pre-training - The optimization view

- ★ Greedily learning one layer at the time scales well to really big networks (especially with locality in each layer)
- ★ We already have sensible feature detectors before backpropagation
- ★ Backpropagation only need to perform local search from a sensible starting point

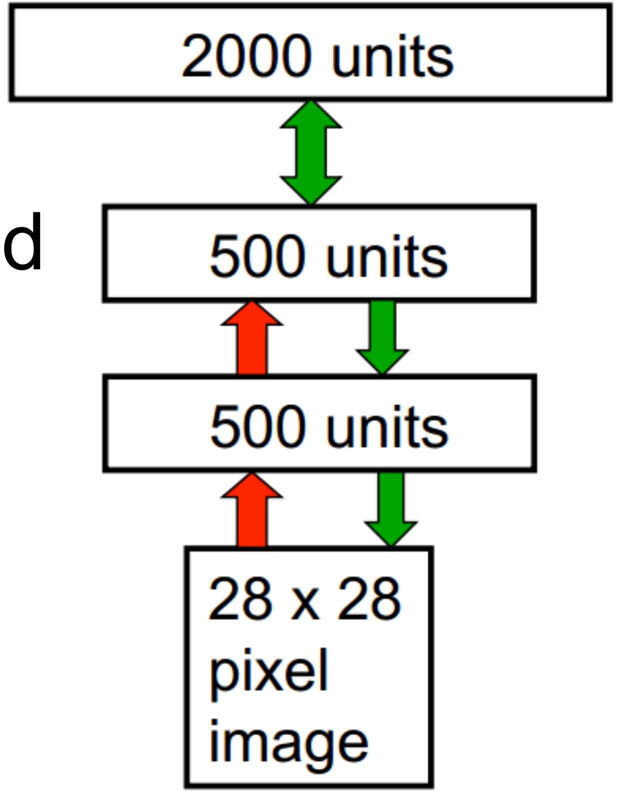
Why backpropagation works better with greedy pre-training - The overfitting view

- ★ Most of the information comes from input vectors (contains more information than the labels)
- ★ Labels are used only for fine-tuning
  - ○ Fine-tuning doesn’t need to discover new features
  - ○ Modifies features slightly to get the class boundaries right
- ★ Works well even for mostly unlabeled data (unlabeled data is still useful for discovering features)

Modelling distribution of digit images

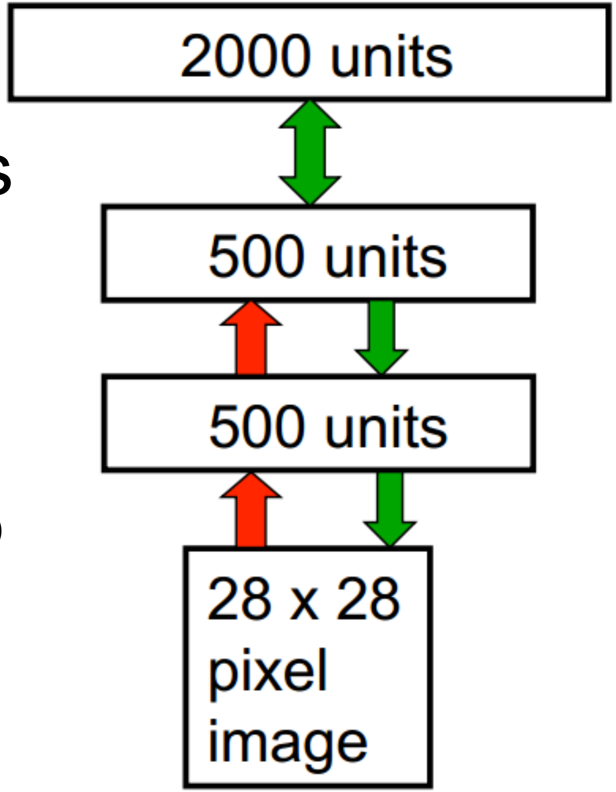
★ The top two layers form a RBM whose energy landscape should model the low dimensional manifolds of the digits

★ Learns density model for unlabeled digit images



Generalization and discrimination of digits

- ★ Generating from the model, we get things that look like real digits of all classes
- ★ Does the hidden features really help with digit discrimination?
  - ○ Add a 10-way softmax at the top
  - ○ Do backpropagation



Results on the permutation-invariant MNIST task

	Error rate
Backprop net with one or two hidden layers (Platt; Hinton)	1.6%

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Generative model of joint density of images and labels (+ generative fine-tuning)	1.25%
Generative model of unlabelled digits followed by gentle backpropagation (Hinton & Salakhutdinov, 2006)	1.15%

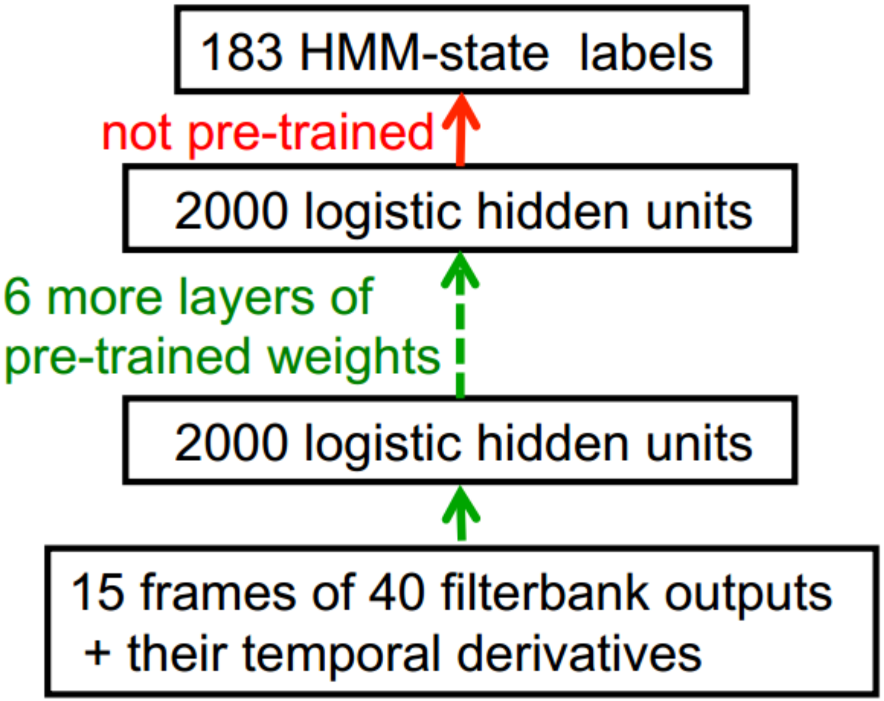
Unsupervised "pre-training" also helps for models with more data and better priors

- ★ Used an additional 600,000 distorted digits
- ★ Also used convolutional multilayer neural networks

Ranzato et. al. (NIPS 2006)

	Error rate
Back-propagation alone	0.49%
Unsupervised layer-by-layer pre-training followed by backpropagation	0.39%

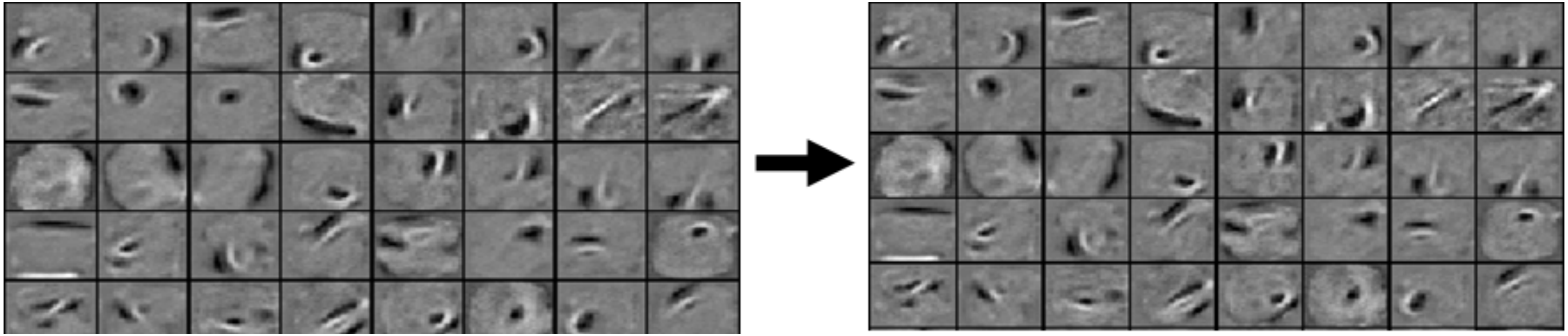
Phone recognition on the TIMIT benchmark



- ★ The best previous speaker-independent result on TIMIT (required averaging multiple models) - 24.4% error rate
- ★ A deep net (after standard post-processing with a bi-phone model) - 20.7%
- ★ Has changed speech recognition

What happens during discriminative fine-tuning?

Learning Dynamics of Deep Nets

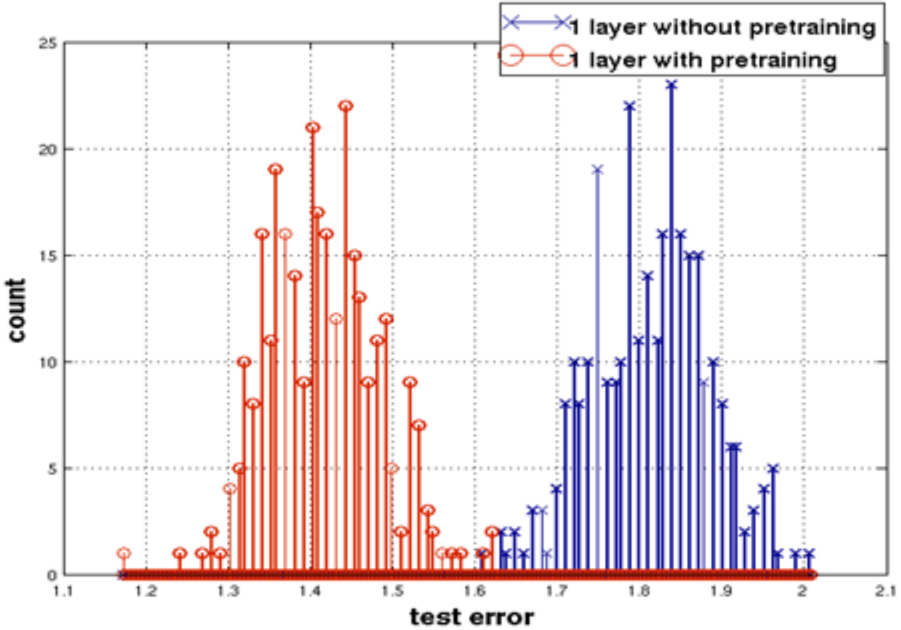


Before fine-tuning

After fine-tuning

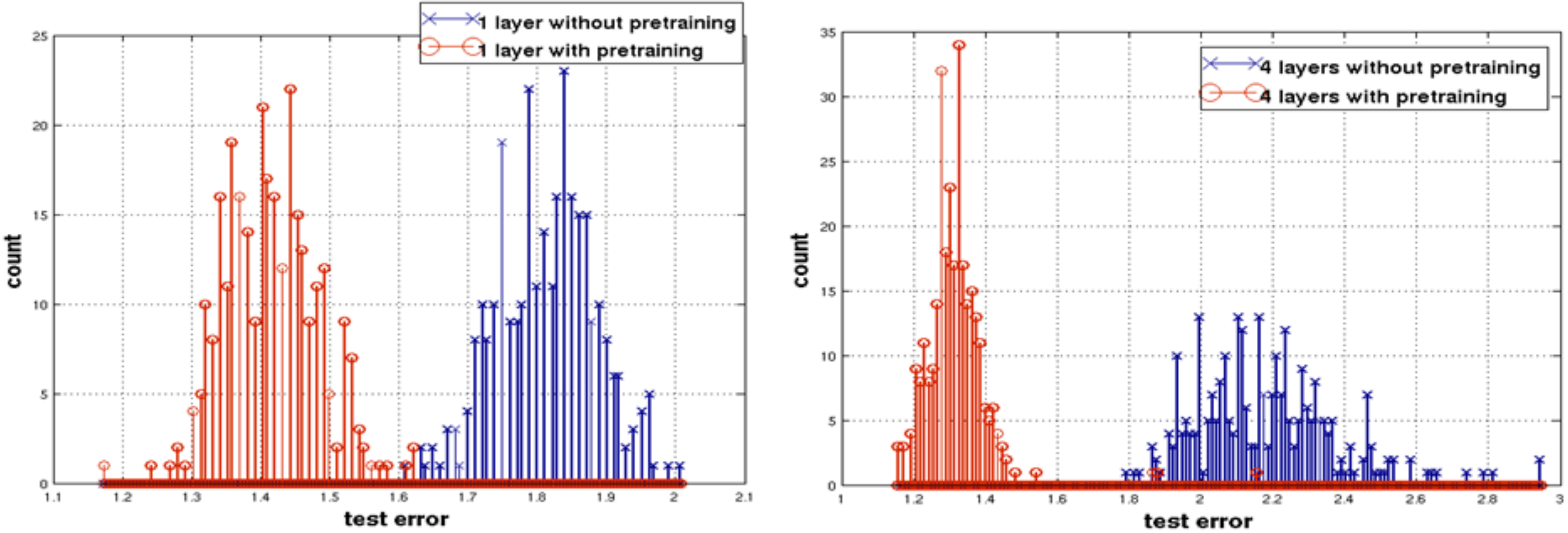
The next 4 slides describe work by Yoshua Bengio's group

Effect of Unsupervised Pre-training



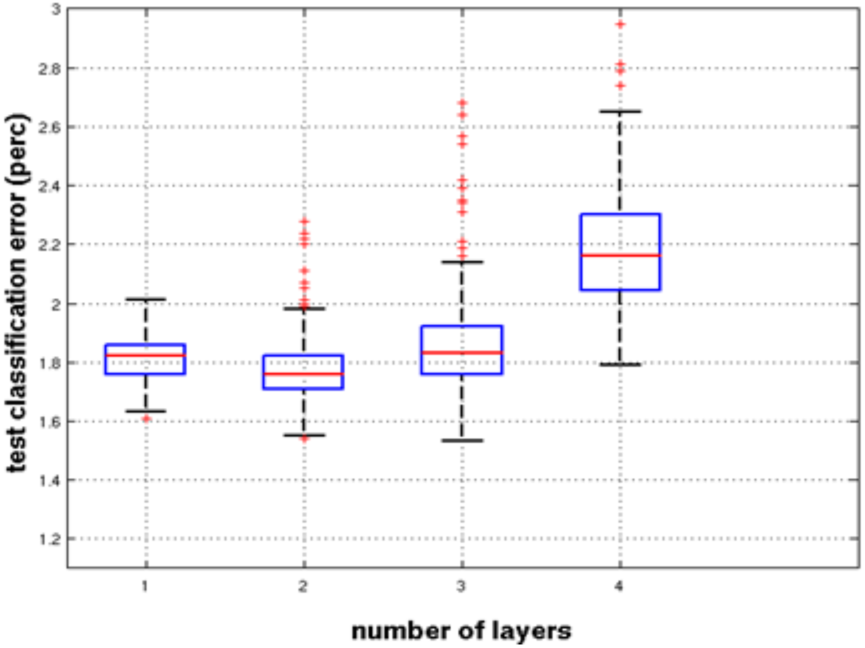
Erhan et. al. AISTATS' 2009

Effect of Unsupervised Pre-training



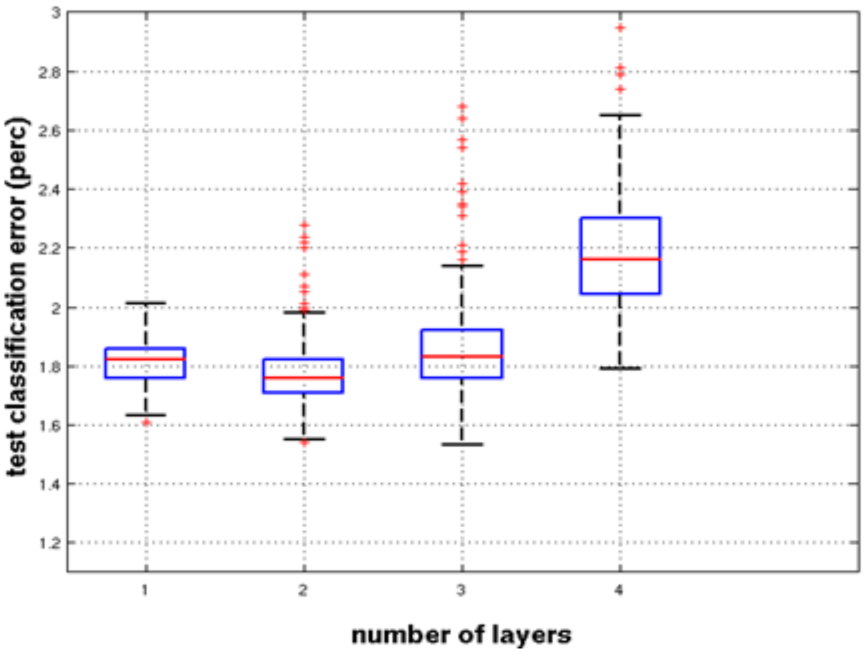
Erhan et. al. AISTATS' 2009

Effect of Depth

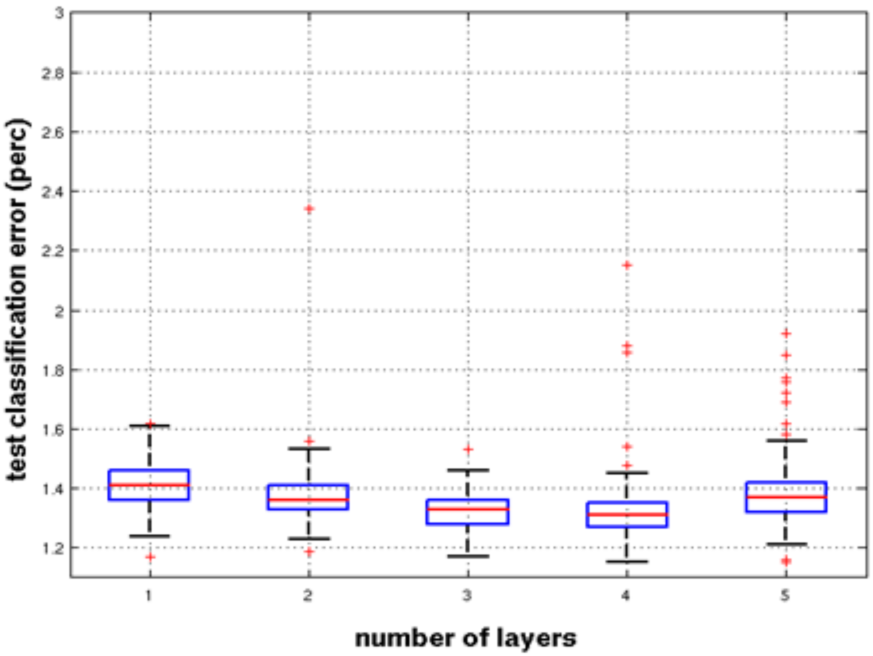


Without pre-training

Effect of Depth



Without pre-training



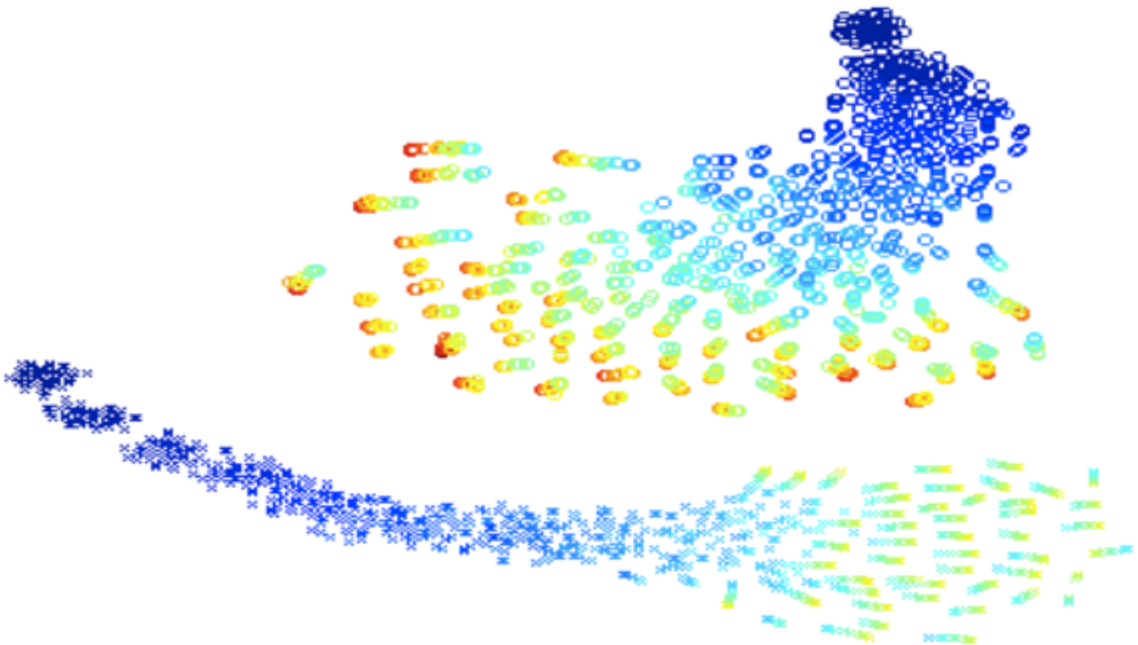
With pre-training

Trajectories of the learning in function space

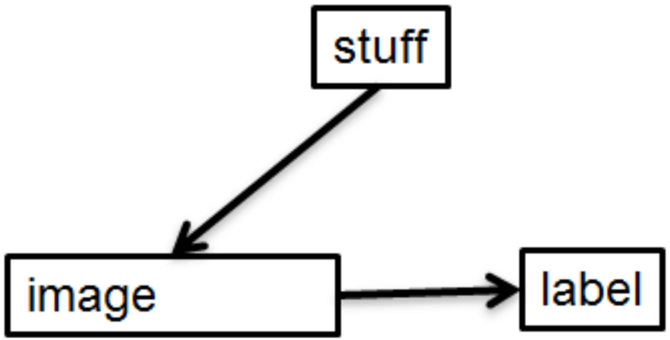
(a 2-D visualization produced with t-SNE)

Erhan et. al. AISTATS' 2009

- ★ Each point is a model in function space.
- ★ Color = epoch
- ★ Top: trajectories without pre-training. Each trajectory converges to a different local minimum.
- ★ Bottom: Trajectories with pre-training.
- ★ No overlap!

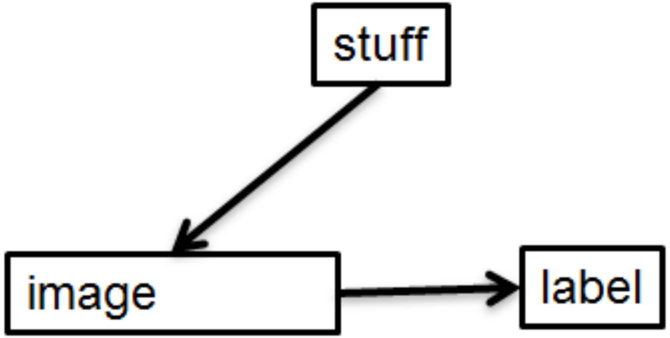


Why unsupervised pre-training makes sense



If image-label pairs were generated this way, it would make sense to try to go straight from images to labels.
For example, do the pixels have even parity?

Why unsupervised pre-training makes sense



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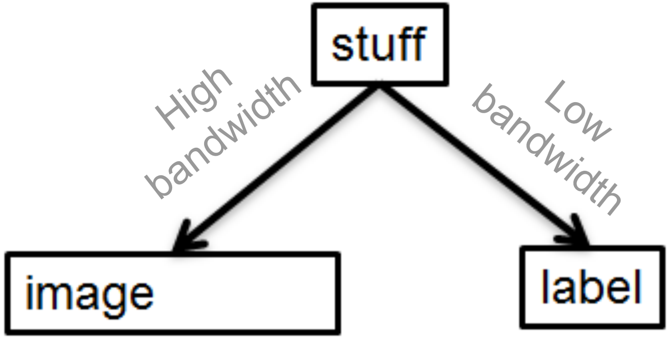


If image-label pairs are generated this way, it makes sense to first learn to recover the stuff that caused the image by inverting the high bandwidth pathway.

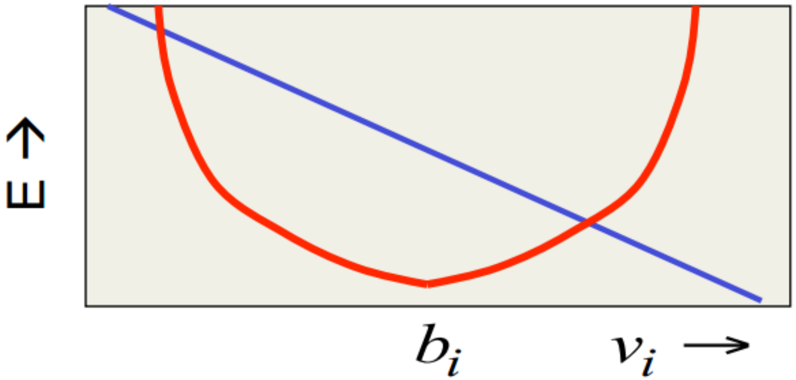
Modeling real-valued data with an RBM

Modeling real-valued data

- ★ For images of digits:
  - ○ intermediate intensities can be represented as probabilities by using “mean-field” logistic units
  - ○ We treat intermediate values as the probability that the pixel is inked
- ★ This doesn’t work for real images:
  - ○ intensity of a pixel is almost always, almost exactly the average of the neighboring pixels
  - ○ Mean-field logistic units cannot represent precise intermediate values

A standard type of real-valued visible unit

- ★ Model pixels as Gaussian variables
- ★ Alternating Gibbs sampling is still easy
- ★ Learning needs to be much slower



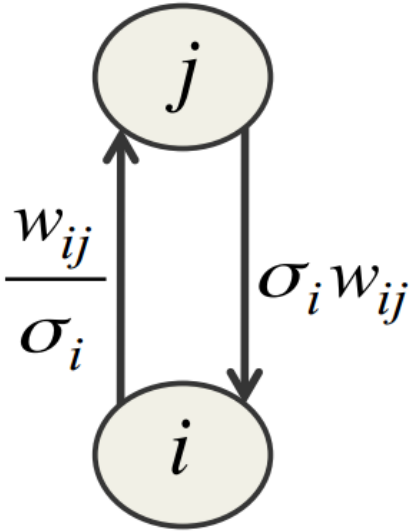
$$E(\mathbf{v},\mathbf{h}) = \sum_{i\,\varepsilon\,vis} \frac{(v_i - b_i)^2}{2\sigma_i^2} - \sum_{j\,\varepsilon\,hid} b_j h_j - \sum_{i,j} \frac{v_i}{\sigma_i} h_j w_{ij}$$

parabolic containment function

energy-gradient produced by the total input to a visible unit

Gaussian-Binary RBM's

- ★ Its extremely hard to learn tight variances for the visible units
- ★ When sigma is small, we need many more hidden units than visible units



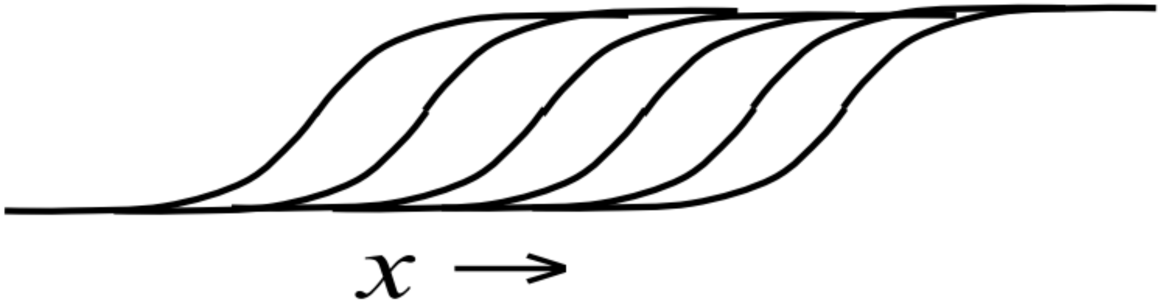
When sigma is much smaller than 1:

- ★ Bottom-up effects are too big
- ★ Dop-down effects are too small

Stepped sigmoid units: implementing integer values

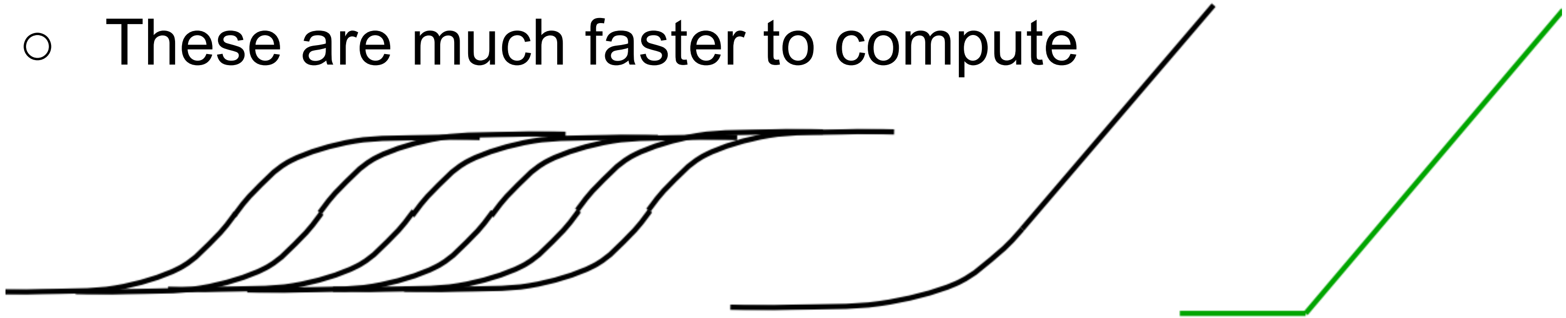
- ★ Make many copies of a stochastic binary unit
  - ○ Same weights
  - ○ Same adaptive bias b
  - ○ Different fixed offsets to bias:

$$b-0.5,\ b-1.5,\ b-2.5,\ b-3.5, \ldots$$



Fast approximations

- ★ Contrastive divergence learning works well for the sum of stochastic logistic units with offset biases.
  - ○ The noise variance is $\sigma(y)$
- ★ Also works for rectified linear units
  - ○ These are much faster to compute



$$\langle y \rangle = \sum_{n=1}^{n=\infty} \sigma(x + 0.5 - n) \quad \approx \quad \log(1 + e^{x}) \quad \approx \quad \max(0, x + noise)$$

A nice property of rectified linear units

★ If a rectified linear unit has a bias of zero, it exhibits scale equivariance

- A very nice property to have for images

  $$R(a\mathbf{x}) = a\,R(\mathbf{x}) \qquad but \qquad R(a+b) \neq R(a) + R(b)$$

- It is like the equivariance to translation exhibited by convolutional nets.

  $$R(shift(\mathbf{x})) = shift(R(\mathbf{x}))$$

THANK YOU!

Thank god it's over!