Simple types
2 Simple types
2.1 Types and typing rules

Types and typing rules
Typed terms

- Types (blackboard).
- Variables and terms (blackboard).
- Type environments (blackboard).
## Typing

- Typings and typing assertions (blackboard).
- Typing rules (blackboard).
- Typing derivations (blackboard).
Evaluation

- Values (blackboard).
- Reduction rules (blackboard).
- Reduction sequences (blackboard).
2 Simple types
2.1 Types and typing rules

Examples

• Find a derivation of the type assertion

\[ f : X \to X \vdash (\forall x : X \cdot f \, x) : X \to X \]

(blackboard).

• Let

\[ W = \forall h : (A \to A) \to (A \to A) \to A \to A \cdot \forall x : A \to A \cdot h \, x \, x, \]
\[ K = \forall x : A \to A \cdot \forall y : A \to A \cdot x, \]
\[ I = \forall z : A \to A \cdot z, \]
\[ I' = \forall z : A \cdot z \]

and let \( t = W \, K \,(I \, I') \). Find a type \( T \) and a derivation of the type assertion

\( \vdash t : T \), and a reduction sequence of \( t \) that ends with a value (blackboard).
Exercises

• Find a derivation of the type assertion $x : X \vdash (\lambda z : X. z) x : X$ (oneself).

• Find a type environment $\Gamma$ such that $\Gamma \vdash f \, x \, y : X$ is derivable (oneself).

• Let

$$W = \lambda h : (A \to A) \to (A \to A) \to A \to A. \backslash x : A \to A. h \, x \, x,$$

$$K = \lambda x : (A \to A) \to A \to A. \backslash y : A \to A. x,$$

$$I = \backslash z : A \to A. z,$$

$$I' = \backslash z : A. z$$

and let $t = W \, (K \, I) \, I'$. Find a type $T$ and a derivation of the type assertion $\vdash t : T$, and a reduction sequence of $t$ that ends with a value (blackboard).
Type erasure
Definitions

• Definition 9.5.1: erasure (blackboard).

• Definition 9.5.3: typability (blackboard).
2 Simple types
2.2 Type erasure

Theorems

- Theorem 9.5.2: Erasure preserves reduction (blackboard).
- Exercise 8.2.3: Subterms of typable terms are typable (oneself).
- Exercise 9.3.2: The term $x x$ is not typable (oneself).
Properties of typing
Basic lemmas

• **Weakening**: If all typings of $\Gamma$ occur in $\Delta$ then derivability of $\Gamma \vdash t : T$ implies derivability of $\Delta \vdash t : T$ (blackboard).

• **Permutation**: If $\Gamma$ is a type environment and $\Delta$ is its permutation then derivability of $\Gamma \vdash t : T$ is equivalent to derivability of $\Delta \vdash t : T$ for every term $t$ and type $T$ (blackboard).

• **Substitution**: If both $\Gamma, x : S \vdash t : T$ and $\Gamma \vdash s : S$ are derivable then $\Gamma \vdash [x \mapsto s] t : T$ is derivable (blackboard).
Uniqueness

- Let \( \Gamma \) be a type environment and \( t \) a term. Then there exists at most one type \( T \) such that \( \Gamma \vdash t : T \) is derivable (blackboard).

- Let \( \Gamma \) be a type environment, \( t \) a term and \( T \) a type. Then there exists at most one derivation of \( \Gamma \vdash t : T \) (blackboard).
Safety

• **Progress**: For any term $t$ and type $T$, if $\vdash t : T$ then either $t$ is a value or $t \Rightarrow_{\beta} u$ for some term $u$ (blackboard).

• **Preservation**: For arbitrary type environment $\Gamma$, terms $t$, $u$ and type $T$, if $t \Rightarrow_{\beta} u$ and $\Gamma \vdash t : T$ is derivable then $\Gamma \vdash u : T$ is derivable (blackboard).
Subject reduction and expansion

• Preservation is often called subject reduction property because $t$ and $T$ in the typing $t : T$ are linguistically a subject and a predicate whence the property states preservation of typing under reduction of the subject (blackboard).

• Does the subject expansion property also hold: For arbitrary type environment $\Gamma$, terms $t$, $u$ and type $T$, if $t \Rightarrow_\beta u$ and $\Gamma \vdash u : T$ is derivable then $\Gamma \vdash t : T$ is derivable (oneself)?
Normalization

• Exercise 12.1.1 (oneself).

• Definition 12.1.2 detailed (blackboard).

• Lemma 12.1.4 (blackboard).

• Lemma 12.1.5 (blackboard).

• Theorem 12.1.6: If $\vdash t : T$ is derivable then $t$ is normalizable (blackboard).
Curry-Howard Isomorphism
Propositional logic with implication

• Propositions (blackboard).
• Sequents (blackboard).
• Derivations (blackboard).
Correspondence between type system and logic

- Types vs propositions, terms vs proofs (blackboard).
- Cut elimination (blackboard).