Big O and Simple Data Structures

Oliver-Matis Lill

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Big O

- Recall that $f(x) \in O(g(x))$ means that:

$$|f(x)| \leq M|g(x)| \text{ for all } x > x_0 \text{ where } x_0 \in \mathbb{R}, M \in \mathbb{R}^+$$

- To prove $f(x) \in O(g(x))$ we just need to find constants x_0 and M where the above holds
- To prove $f(x) \notin O(g(x))$ we can show that for each M and x_0 there exists some $x > x_0$, such that $|f(x)| > M|g(x)|$

Example

Let's show that:

$$2x^3 + 5x^2 + 4x + 9 \in O(x^3)$$

1. Note that $x^3 > x^2 > x > 1$ for all $x > 1$. Pick $x_0 = 1$
2. Let's pick $M = 2 + 5 + 4 + 9$, then we have $M|g(x)| = 2x^3 + 5x^3 + 4x^3 + 9x^3$
3. By combining (1) and (2) we see that $|f(x)| \leq M|g(x)|$ for $x > x_0$ given our picks of x_0 and M

Example

Let's show that:

$$2x^3 + 5x^2 + 4x + 9 \notin O(x^2)$$

1. Note that $f(x) > x^3$ for all $x > 1$
2. Suppose by contradiction that there exist x_0 and M satisfying the condition
3. Pick $x = cM$, where c is a positive integer such that $cM > x_0$
4. Combining (1) and (3) we get that $|f(x)| > c^3M^3 \geq c^2M^3 = M|g(x)|$, which contradicts (2)

Problem

Given that:

1. $f_1(n) \in O(g_1(n))$
2. $f_2(n) \in O(g_2(n))$
3. $g_1(n) \in O(g_2(n))$

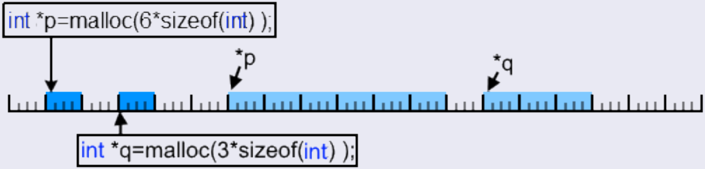
Prove that:

$$f_1(n) + f_2(n) \in O(g_2(n))$$

Memory Recap

- Recall that an array stores it's data consecutively:

- Pointer is a variable that points to a memory location
- You can allocate variable amount of memory on runtime and use pointers to refer to it:



Memory Recap

Exercise

Look at two implementations of a 2-D array:

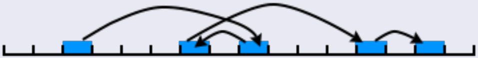
```
// Implementation A
int **a = malloc(8*sizeof(int*));
for(int i = 0; i < 8; i++) {
    a[i] = malloc(11 * sizeof(int));
}
```

```
// Implementation B
int a[8][11];
```

1. How would memory be arranged in implementation A?
2. In A, how many memory locations would you have to read to get the value of some element a[i][j]? How many in B?
3. What are the advantages and disadvantages of A and B?

Lists

- A list is a group of objects connected into a "chain" using pointers. It might look like this:



- It's advantage is that it allows you to insert to and delete from anywhere in $O(1)$
- On the flip side the elements are dispersed in memory, eliminating vectorization and cache locality opportunities and making it much slower than an array in practice

List Code

Example

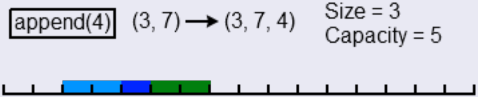
```
void insert(struct List* list, struct Node* pos, int toAdd) {
    struct Node* cur = malloc(sizeof(struct Node) );
    cur->prev = pos; cur->value = toAdd;
    if(cur->prev == 0) {
        cur->next = list->first;
        list->first = cur;
    }
    else {
        cur->next = cur->prev->next;
        cur->prev->next = cur;
    }
    if(cur->next == 0)
        list->last = cur;
    else
        cur->next->prev = cur;
}
```

Exercises

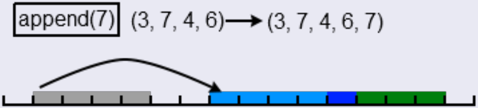
- What are the differences between a list that allows traversal in only one direction and another that allows both directions?
- How does a list of 32-bit integers look like in memory in a 32-bit program? What is the memory overhead?
- Suppose we want to get the i-th element. How many operations do we have to do asymptotically (in Big O)? How does it compare to an array?

Vectors

- Vectors are basically resizeable arrays
- Vectors have a capacity. They reserve some extra memory where new elements are added



- Once capacity is exceeded the following is done:
  1. A new memory block with (2x) larger capacity is allocated
  2. The old data is copied over into the new block
  3. The old block is deallocated



Vectors

- $O(1)$ append operation (insertion to the last position)
- The advantage is that the data is consecutive, so in practice its about as fast as an array with the benefit of resizeability
- The disadvantage is that it can have significant memory overhead (up to 3x during reallocation), however usually it doesn't matter
- Note that internally vector is just some metadata with a pointer to the data. What's interesting is how the metadata is used to facilitate resizeability

Vector Code

Example

```
void append(struct Vector* vector, int toAdd) {
    if(vector->size == vector->capacity) {
        int* newData = malloc(2*vector->capacity*sizeof(int) );
        memcpy(newData, vector->data, vector->size * sizeof(int));
        free(vector->data);
        vector->data = newData;
        vector->capacity *= 2;
    }
    vector->data[vector->size++] = toAdd;
}
```

Vectors

Exercises

- Suppose we create pointers that point to elements of a vector. What are the risks of doing that?
- Suppose we fill a vector by appending m times. How many operations do we have to do asymptotically (in Big O)?
- Suppose we create a vector of vectors of integers (vector<vector<int> >). Suppose that:
  1. We add m integers in total to subvectors
  2. All subvectors will end up non-empty
  3. The subvectors can be added at any time

  What is the worst case complexity (Big O of the number of operations) if in the top-level vector reallocation we:
  1. Move only the metadata of the lower-level vectors
  2. Move the whole contents of the lower-level vectors

Deques

- Deques are resizeable sequences like vectors, but internally they behave differently
- Deques store elements in (reasonably large) chunks and when capacity is exceeded they simply add a new chunk

- The data is more consecutive than in a list, but less than in a vector
- As a result in practice it's much faster than a list, but somewhat slower than a vector
- The main benefit over a vector is that no reallocation is performed, so pointers to its data don't get invalidated