Algorithmics (6EAP)
Search and meta-heuristics – Local search

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Search Methods

• Types of search methods:
  • systematic $\leftrightarrow$ local search
  • deterministic $\leftrightarrow$ stochastic
  • sequential $\leftrightarrow$ parallel
Local Search
Local search algorithms

• In many optimization problems, the path to the goal is irrelevant; the goal state itself is the solution.

• State space = set of "complete" configurations
  • Find configuration satisfying constraints, e.g., n-queens

• In such cases, we can use local search algorithms
  • keep a single "current" state, try to improve it
Example: $n$-queens

- Put $n$ queens on an $n \times n$ board with no two queens on the same row, column, or diagonal.

- may get stuck...
Problems

• Problem: Cycles
  – Memorize
  – Tabu search

• How to transfer valleys with bad choices only...
Local Search (LS) Algorithms

• **search space** $S$
  (SAT: set of all complete truth assignments to propositional variables)

• **solution set** $S' \subseteq S$
  (SAT: models of given formula)

• **neighborhood relation** $N \subseteq S \times S$
  (SAT: neighboring variable assignments differ in the truth value of exactly one variable)

• **evaluation function** $g : S \rightarrow \mathbb{R}^+$
  (SAT: number of clauses unsatisfied under given assignment)
Local Search:

• start from initial position
• iteratively move from current position to neighboring position
• use evaluation function for guidance

Two main classes:

• local search on partial solutions
• local search on complete solutions
Local search for partial solutions

- Order the variables in some order.
- Span a tree such that at each level a given value is assigned a value.
- Perform a depth-first search.
- But, use heuristics to guide the search. Choose the best child according to some heuristics. (*DFS with node ordering and backtracking*)
Construction Heuristics for partial solutions

• search space: space of partial solutions
• search steps: extend partial solutions with assignment for the next element
• solution elements are often ranked according to a greedy evaluation function
The Traveling Salesperson Problem (TSP)

- **TSP – optimization variant:**
  - For a given weighted graph $G = (V,E,w)$, find a Hamiltonian cycle in $G$ with minimal weight,
  - i.e., *find* the shortest round-trip visiting each vertex exactly once.

- **TSP – decision variant:**
  - For a given weighted graph $G = (V,E,w)$, *decide* whether a Hamiltonian cycle with minimal *weight* $\leq b$ exists in $G$. 
Nearest Neighbor heuristic for the TSP:

- at any city, choose the closest yet unvisited city
  - choose an arbitrary initial city $\pi(1)$
  - at the $i$th step choose city $\pi(i+1)$ to be the city $j$ that minimises $d(\pi(i), j); j \neq \pi(k), 1 \leq k \leq i$

- running time: $O(n^2)$

- worst case performance:
  $\frac{\text{NN}(x)}{\text{OPT}(x)} \leq 0.5([\log_2 n] + 1)$

- other construction heuristics for TSP are available
Nearest neighbor tour through 532 US cities
Figure 114: The solid minimum spanning tree, the dotted traversal using each edge of the tree twice, and the solid tour obtained by taking short-cuts.

https://www.cs.duke.edu/courses/fall08/cps230/Lectures/L-25.pdf
TSP instance: shortest round trip through 532 US cities
My current best is 27486.199404966355 (nn gives 27766.484757657887)

All the best,

Polina
My best is 24839,308924381 (Jaak S)
My new best is 23474 (Oleg)
23297.72476804589
Probably some local minimum near Jaak Sarv's solution
A shortest-possible walking tour through the pubs of the UK

A shortest-possible walking tour through the pubs of the United Kingdom — that’s an advanced form of the mathematicians’ favorite, The Traveling Salesman Problem. William Cook and colleagues at the University of Waterloo tackled this nastily complex problem:

Nearly everyone in the UK knows by heart the best path to take them over to their favorite public house. But what about jotting down the shortest route to visit every pub in the country and return home safely? That is what we set out to do.

Using geographic coordinates of 24,727 pubs provided by Pubs Galore and measuring the distance between any two pubs as the length of the route produced by Google Maps, what is the shortest possible tour that visits all 24,727 and returns to the starting point? …

This is the problem we have solved. The optimal tour has length 45,495,239 meters. To be clear, our main result is that there simply does not exist any pub tour that is even one meter shorter (measuring the length using the distances we obtained from Google) than the one produced by our computation. It is the solution to a 24,727-city traveling salesman problem (TSP).

The UK Pubs tour is easily the largest such road-distance TSP that has been solved to date, having over 100 times more stops than any road-distance example solved previously by other research groups.

Here’s one low-resolution sliver of what is a much more detailed map of the tour:

History of TSP:
http://www.math.uwaterloo.ca/tsp/
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