Advanced Algorithmics (6EAP)

courses.cs.ut.ee

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Introduction

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Programming on one slide

• TRUE/FALSE, boolean operations
• Data: 3, 7, 2.47, .333e-02, ‘a’, “text data”, address ...
• Operations: + - * / > <= ...
• Variables: i, j, sum, A[5], B[7,8], a[i].surname, ...
• if-then-else, for/while, ...
• function()
• Input/output
• Standard libraries: java.lang.Math.sqrt()

Goals

• To learn the main concepts and techniques of the algorithm and data structure design and analysis – the practical skills and basic theoretical basis
• To be able to choose, (design,) analyze and compare algorithms and data structures
• To learn to learn, use wisely, solve, read, write, and present

Algorithms = basics of CS education

• In your future professional life people EXPECT you to know elementary data structures, algorithms, and be able to think in higher categories.
• We need to develop the language to talk about high-level goals, and you need to be able to implement them – if so needed
• At work no-one is going to teach you mathematics or fundamentals anymore

Algorithms

• Al-Khwārizmī

• Abū Abdallāh Muhammad ibn Mūsā al-Khwārizmī

Example

• Wind has blown away the +, *, (, ) signs
• What’s the maximal possible value?
• Minimal?

2 1 7 1 4 3
• \(2 \ 1 \ 7 \ 1 \ 4 \ 3\)

\[(2+1) \times 7 \times (1+4) \times 3 = 21 \times 15 = 315\]

\[2 \times 1 + 7 + 1 \times 4 + 3 = 16\]

• Q: How to maximize the value of any expression?

\[2 \ 4 \ 5 \ 1 \ 9 \ 8 \ 12 \ 1 \ 9 \ 8 \ 7 \ 2 \ 4 \ 4 \ 1 \ 1 \ 2 \ 3 = ?\]

School mathematics: 11 \times 13

But Al Khowarizmi knew another way to multiply: a method which is still used today in some European countries. To multiply two decimal numbers \(x\) and \(y\), write them next to each other, as in the example below. Then repeat the following: divide the first number by 2, rounding down the result if that is, dropping the \(\frac{1}{2}\) if the number was odd, and double the second number. Keep going till the first number gets down to 1. Then strike out all the rows in which the first number is even, and add up whatever remains in the second matrix.

\[
\begin{array}{c}
11 \\
5 \\
2 \\
1
\end{array}
\times
\begin{array}{c}
13 \\
26 \\
52 \\
104
\end{array}
= 143 \text{ (answer)}
\]

13 \times 11

\[
\begin{array}{c}
1 \\
1 \\
1 \\
0 \\
1 \\
0 \\
1 \text{ (binary 143)}
\end{array}
\times
\begin{array}{c}
1 \\
0 \\
0 \\
1 \\
1 \\
1 \\
1 \\
1 \\
1 \\
1 \\
1 \\
1 \\
1 \text{ (binary 143)}
\end{array}
\]

\[
\begin{array}{c}
1 \\
1 \\
0 \\
1 \text{ (1101 times 1)}
\end{array}
\times
\begin{array}{c}
1 \\
1 \\
0 \\
1 \\
0 \\
0 \\
0 \\
0 \\
1 \\
1 \\
0 \\
1 \text{ (1101 times 0, shifted twice)}
\end{array}
\]

\[
\begin{array}{c}
1 \\
1 \\
0 \\
1 \text{ (1101 times 1, shifted once)}
\end{array}
+ \begin{array}{c}
1 \\
1 \\
0 \\
1 \text{ (1101 times 0, shifted twice)}
\end{array}
\]

\[
\begin{array}{c}
1 \\
0 \\
0 \\
1 \\
1 \\
1 \\
1 \\
1 \\
1 \text{ (binary 143)}
\end{array}
\]

Analysis of algorithms

• Theoretical study of computer-program performance and resource usage.

• What else is important than performance? e.g.

  • Modularity
  • Correctness
  • Maintainability
  • Functionality
  • Robustness
  • User-friendliness
  • Programmer time
  • Simplicity
  • Extensibility
  • Reliability
Why study algorithms and performance?

• Algorithms help us to understand **scalability**.
• Performance often draws the line between what is feasible and what is **impossible**.
• Algorithmic mathematics provides a **language for talking about program behavior**.
• Performance is the **currency of computing**.
• The lessons of program performance generalize to other computing resources.
• Speed is fun!

Steve Jürvetson on Skype:

• “I remember wondering: how can they be so good?” he told me, speaking about the Estonian core of Skype. “How can such a small group can do so much so quickly, compared to typical development efforts in, for example, Microsoft? I had the impression that maybe coming out of a time of Soviet occupation, when computers were underpowered, you had to know how to really program, effectively, parsimoniously, being very elegant in sculpting the programming code to be tight, effective, and fast. [That’s] not like in Microsoft, which has a very lazy programming environment, where programs are created that have memory leaks and all sorts of problems, that crash all the time and no one really cares—because it’s Microsoft!”


Contents

• Algorithms and Data structures – basics
• Abstract data types (libraries, templates)
• Analysis of algorithm complexity
  – \( O(n \log n) \), \( o(n \log n) \), \( \Theta(n \log n) \), \( \Omega(n \log n) \)
  – recurrences \( T(n) = 2T(n/2) + n \)
  – amortised analysis
• Basic designs: **Divide and conquer, dynamic programming, randomised algorithms, …**

Contents (2)

• **Data structures**: Lists, trees, heaps, graphs, …
• Sorting, searching, dictionary, …
• Text algorithms
  – Search: KMP, Aho-Corasick, Boyer-Moore
  – indexing – suffix trees, suffix arrays
• Implicit and succinct data structures
• Augmenting data structures examples
• Combinatorial optimization and search

Contents (3)

• Heuristics (approximations):
  – **Greedy** Algorithms (packing, set cover, weighted set cover, …) local search, tabu search
  – **Simulated annealing** (e.g. TSP),
  – Genetic algorithms (?), … evolutionary strategies
  – Differential Evolution, Ant Colony, Particle Swarm …

Skills/goals

• High level understanding (theory)
• Bit-level understanding of low-level implementation
• Creative use of existing implementations
• Formal analysis, complexity theory
• Typical heuristics and algorithm design elements
• 67 + 56 = ?

SUM( 10 7 98 82 84 66 62 94 6 25 73 62 65 5 36 983 73 93 77 10 7 98 82 84 66 62 94 6 25 73 62 65 5 36 983 73 93 77 10 7 98 82 84 66 62 94 6 25 73 62 65 5 36 983 73 93 77 10 7 98 82 84 66 62 94 6 25 73 62 65 5 36 983 73 93 77 10 7 98 82 84 66 62 94 6 25 73 62 65 5 36 983 73 93 77 10 7 98 82 84 66 62 94 6 25 73 62 65 5 36 983 73 93 77 10 7 98 82 84 66 62 94 6 25 73 62 65 5 36 983 73 93 77 10 7 98 82 84 66 62 94 6 25 73 62 65 5 36 983 73 93 77 10 7 98 82 84 66 62 94 6 25 73 62 65 5 36 983 73 93 77 10 7 98 82 84 66 62 94 6 25 73 62 65 5 36 983 73 93 77 10 7 98 82 84 66 62 94 6 25 73 62 65 5 36 983 73 93 77 10 7 98 82 84 66 62 94 6 25 73 62 65 5 36 983 73 93 77 10 7 98 82 84 66 62 94 6 25 73 62 65 5 36 983 73 93 77 10 7 98 82 84 66 62 94 6 25 73 62 65 5 36 983 73 93 77 10 7 98 82 84 66 62 94 6 25 73 62 65 5 36 983 73 93 77 10 7 98 82 84 66 62 94 6 25 73 62 65 5 36 983 73 93 77 10 7 98 82 84 66 62 94 6 25 73 62 65 5 36 983 73 93 77 ) = ?

Answer

• 13,000

• Is checking faster than finding the solution?

SAT satisfiability

• Can a boolean formula with variables instantiated to become “TRUE”
  – ( A or B ) and ( not A or B )

• Check if (some) answer satisfies the formula
  • very quick as compared to finding an answer
Subset sum

- Does some subset of a set sum up to 0?
  - Verifier/witness/certificate
  - Easy to check if someone points out ...
    - \( \{ 2, -4, 3, 8, -1, 9, -6 \} \)
- If verification in P time, then problem in NP
- Complementary: prove that none exists!
  - co-NP
  - Every subset is non-0
All fitted on **linear** RAM, disk, tape

![Linear Tree Graph Relational](image)

- Linear storage: address of a "slot"
- Programming languages:
  - Pseudocode
    - primarily this course will use pseudocode
  - Pointers, arrays, memory handling
    - Low- and high level abstractions
  - C/C++
  - Java, Python, perl, go, ...
    - Explicit data structures and algorithms
  - Whichever choice: you must be able to explain it
  - Clarity and simplicity is a key

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“The Textbook”

- **Introduction to Algorithms**, Second Edition
  - Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest and Clifford Stein
  - [http://mitpress.mit.edu/algorithms/](http://mitpress.mit.edu/algorithms/)

- **Foundations of Computer Science: C Edition**
  - Alfred V. Aho, Jeffrey D. Ullman
  - W. H. Freeman (October 15, 1994)

  - David Harel, Yishai Feldman
  - Addison Wesley; 3 edition (June 11, 2004)
  - [http://books.google.com/books/about/Algorithmics.html?id=txxLovFWkCUC](http://books.google.com/books/about/Algorithmics.html?id=txxLovFWkCUC)

- **The Art of Computer Programming (TAOCP)**
  - Donald E. Knuth.
The Algorithm Design Manual
  – Steven S. Skiena

Algorithms [ILLUSTRATED]
  – Sanjoy Dasgupta, Christos Papadimitriou, Umesh Vazirani

Mathematics
  • Concrete Mathematics: A Foundation for Computer Science (2nd Edition) (Hardcover)
    – Ronald L. Graham, Donald E. Knuth, Oren Patashnik
    • Hardcover: 672 pages
    • Addison-Wesley Professional;
      2 edition (March 10, 1994)

  • Robert Sedgewick
  • Safari Bookshelf (UT)

More books
  • Parallel algorithms
    – An Introduction to Parallel Algorithms Joseph Jaja
  • Heuristics
    – Edition: 2, illustrated Published by Springer, 2008

• Randomized algorithms
• Complexity theory
  This book is a classic, developing the theory, then cataloguing many NP-Complete problems.

Wikipedia:
  • http://en.wikipedia.org/wiki/Algorithm
  • http://en.wikipedia.org/wiki/List_of_algorithm_general_topics
  • http://en.wikipedia.org/wiki/List_of_algorithms
  • http://en.wikipedia.org/wiki/List_of_data_structures
  • http://en.wikipedia.org/wiki/Computational_complexity_theory
  • http://en.wikipedia.org/wiki/List_of_NP-complete_problems
  • http://en.wikipedia.org/wiki/Randomized_algorithm
  • http://en.wikipedia.org/wiki/Parallel_algorithm
  • ...