1. **Bayesian network example**

*Alarm story:* I have a burglar alarm installed at home. It is fairly reliable at detecting a burglary, but also responds on occasion to minor earthquakes. I also have two neighbors, John and Mary. They have promised to call me at work when they hear the alarm. John always calls when he hears the alarm, but sometimes confuses the telephone ringing with the alarm and calls then, too. Mary likes rather loud music and sometimes misses the alarm altogether.

*Five random variables:* A: Alarm; B: Burglary; E: Earthquake; J: JohnCalls; M: MaryCalls (These are binary variables)

(a) Draw the BN structure for the given situation.

(b) We want to define a full probability distribution for this situation.
   i. For every node in the graph, write out the corresponding conditional probability tables (just the structure, without any numbers).
   ii. How many probabilities need to be specified for these conditional probability tables? How many probabilities would need to be given if the same joint probability distribution were specified in a joint probability table?

(c) Which one of the two Bayesian networks given below makes independence assumptions that are not true?

(d) What is the probability that the alarm has sounded and there was a burglary but no earthquake, and both Mary and John call? (The values for CPTs are shown in the lecture slides)

(e) Given that the alarm sounded and there was no earthquake, and both Mary and John call, would you predict that there is a burglar in your home? *From the lecture we know the joint probability* $P(J, M, A, \neg B, \neg E) = 0.00062$
2. Hidden Markov Models (HMM)

We have one Boolean state variable that can have two values: \textit{AtmosphericPressure} = \{Low, High\}. We cannot observe it directly, but we can observe whether it’s raining or not – so evidence is also a Boolean variable: \textit{Weather} = \{Rain, Dry\}.

The following tables specify the initial probabilities of states, transition probabilities and emission/observation probabilities, correspondingly:

\[
\begin{array}{c|cc|c}
\text{State} & P(S_1) & S_2 & P(S_2|S_1) \\
\hline
\text{Low} & 0.4 & \text{Low} & 0.3 \\
& & \text{High} & 0.7 \\
\text{High} & 0.6 & \text{Low} & 0.2 \\
& & \text{High} & 0.8 \\
\end{array}
\]

\[
\begin{array}{c|cc|c}
\text{S} & \text{O} & P(O|S) \\
\hline
\text{Low} & \text{Rain} & 0.6 \\
& \text{Dry} & 0.4 \\
\text{High} & \text{Rain} & 0.4 \\
& \text{Dry} & 0.6 \\
\end{array}
\]

The HMM (for 3 days) looks like the following graph.

(a) Given an observation sequence (Dry, Rain, Rain), compute the most likely hidden state sequence.

\textit{Hint: Lecture slide 80}

(b) Given the history of evidence, (Dry, Rain, Rain), what is the probability that the atmospheric pressure was Low on the second day?

\textit{Note: The same HMM can also be represented as in the figure below.}