MTAT 03.227 Machine Learning
Practice Session 4
SVM

September 30th - October 3rd 2019

1

Make an illustration with the points from Table 1. Try to manually guess what should be the support vectors. Draw the support vector lines and the separating hyperplane (by hand, approximate lines).

	x_1	x_2	y
1	1	1	-1
2	1	3	-1
3	2	5	-1
4	4	2	-1
5	5	5	1
6	5	9	1
7	6	1	-1
8	7	6	1
9	9	1	1
10	11	3	1

Table 1: SVM data

2

Task: Find the weight vector w and bias b of the separating hyperplane and represent it in the form:

$$w_1x_1 + w_2x_2 + b = 0. \tag{1}$$

Hint: In order to find this line, first find one of the lines for the support vector lines. In general, formula for finding the line from two points (a_1, b_1) and (a_2, b_2) is the following:

$$y - b_1 = \frac{b_2 - b_1}{a_2 - a_1}(x - a_1). \tag{2}$$

Then you can take two support vectors, one on each support vector line and find their mean point. That point has to be on the separating hyperplane. Since the separating hyperplane is parallel to the already found line, their w values are equal, but bias is different. You can plug in the found point to the formula and find the right value for bias b.

3

We represent $f(x) = wx + b$. At the moment the value of function f in the support vectors is -1.5 and 1.5. This is completely ok but in order to make future explanations more elegant and to make the optimization task for finding the suitable weight vector easier to write down, we can scale the weight vector and bias so that the support vectors would have function value of 1.

Task: Find the scaled w and b. You can do this by dividing the line formula by 1.5. Why?

4

There are two types of margins we can talk about. Functional margin and geometrical margin. Functional margin shows the function value of some point. Geometrical margin shows the actual distance of the point to the separating hyperplane. When we do scaling of the weights and bias as in task 3 then the functional margin will change (because the function changes), but geometrical margin stays the same (because the geometrical position of the separating hyperplane doesn't change with the scaling). Definitions of the two margins are:

$$\text{Functional margin: } y_i f(x_i) \tag{3}$$

$$\text{Geometrical margin: } \frac{y_i f(x_i)}{||w||} \tag{4}$$

This comes directly from the formula for finding point distance from line, except that the upper part is multiplied with the true label to achieve a sign for the margin indicating correctness of classification.

Task: Find the functional and geometrical margin for the 4th point. We know that the geometrical margin shows the actual distance from point to line. But what does the functional margin show? What can you say about when the functional margin is positive and when it is negative?

5

At the moment we can use hard-margin SVM to find a separating hyperplane because the points are separable. But let's add a datapoint $((9,4),-1)$. Now the data is not linearly separable. We would need to use soft-margin SVM that tries to find the best line by allowing mistakes, but penalizing them. The penalty is $\xi_i = max(0, 1 - y_i f(x_i))$.

Task: Find the ξ_i for the new point. What does it represent on the plot?

Hint (task 1): support vectors are 5th, 7th and 9th point.