Concurrent Systems Modeling using Petri Nets – Part II

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(Based on lecture material by Wil van der Aalst
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http://www.workflowcourse.com)

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Classical Petri nets - Recap

• Place: passive element
• Transition: active element
• Arc: causal relation
• Token: elements subject to change

The state (space) of a Petri net is a distribution of tokens across its places.
Transition firing move the net from one state to another.
Today’s agenda

1. Patterns of Petri net modeling
2. Expressiveness limitations of Petri nets
3. Introduction to Petri net analysis

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Role of a token

Tokens can play the following roles:

- a **physical object**, for example a product, a part, a drug, a person;
- an **information object**, for example a message, a signal, a report;
- a **collection of objects**, for example a truck with products, a warehouse with parts, or an address file;
- an **indicator of a state**, for example the indicator of the state in which a process is, or the state of an object;
- an **indicator of a condition**: the presence of a token indicates whether a certain condition is fulfilled.
Role of a place

- a type of communication medium, like a telephone line, a middleman, or a communication network;
- a buffer: for example, a depot, a queue or a post bin;
- a geographical location, like a place in a warehouse, office or hospital;
- a possible state or state condition: for example, the floor where an elevator is, or the condition that a specialist is available.
Role of a transition

• an **event**: for example, starting an operation, the death of a patient, a change seasons or the switching of a traffic light from red to green;

• a **transformation of an object**, like adapting a product, updating a database, or updating a document;

• a **transport of an object**: for example, transporting goods, or sending a file.
Typical Petri net structures

• Causality
• Parallelism (AND-split - AND-join)
• Choice (XOR-split – XOR-join)
• Iteration (XOR-join - XOR-split)
• Capacity constraints
  – Feedback loop
  – Mutual exclusion
  – Alternating
Causality
Parallelism

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Parallelism: AND-split
Parallelism: AND-join
Choice: XOR-split
Choice: XOR-join
Woped short-cuts for XOR-split/join
Iteration: 1 or more times

XOR-join before XOR-split

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Capacity constraints: feedback loop
Capacity constraint: example

The room has a capacity of 2, 1 person is in the room
Capacity constraint: exercise

- Pipelined factory
- Two machines, one robot, one buffer
- Robot moves parts from raw line to M1 to buffer, to M2 and finally to finished line
- Buffer can hold maximum 7 parts

Inspired by an exercise of Prof. R. Wattenhofer, ETH Zürich
Capacity constraints: mutual exclusion

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Capacity constraints: alternating
Example: Mutual exclusion with alternation

How to make them alternate?

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Not everything is possible

- Petri nets are not Turing complete
  - Otherwise we wouldn’t be able to fully analyze them (see later)
- Limitations
  - Zero testing
  - Priority
Modeling problem (1): Zero testing

- Transition $t$ should fire if place $p$ is empty.
Solution

• Only works if place is N-bounded

Initially there are N tokens

N input and output arcs

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Modeling problem (2): Priority

- Transition t1 has priority over t2

Hint: similar to Zero testing!
Theoretical Results

- Extensions have been proposed to tackle these problems, e.g., inhibitor arcs.
- These extensions extend the modeling power up to Turing completeness.
- Without such an extension not Turing complete.
- Still certain questions are difficult/expensive to answer or even undecidable (e.g., equivalence of two nets).
Exercise: Witness statements

• As part of the process of handling insurance claims, we need to handle witness statements.

• There may any number of witnesses per claim. The number is determined when the claim is lodged. After an initialization step (one per claim), each of the witnesses is registered and contacted (N witnesses per claim in parallel). Only after all witnesses have contacted a report is made (one report per claim).

• Can you model this using a Petri net?
Quick Intro to Petri net analysis: Reachability Graph

• Graph containing one node for each reachable state; an edge indicates that the system can move from the source state to the target state through one transition firing

• The reachability graph can be calculated as follows:

1. Let $X$ be the set containing just the initial state and let $Y$ be the empty set.
2. Take an element $x$ of $X$ and add this to $Y$. Calculate all states reachable for $x$ by firing some enabled transition. Each successor state that is not in $Y$ is added to $X$.
3. If $X$ is empty stop, otherwise goto 2.
Example: How does this game end?

- Hint: initially the game is in state “3 red + 2 blacks” and there are three transitions from this initial state: rr, rb and bb. Transition “rr” leads to state “1 red + 3 blacks”.
Nodes in the reachability graph can be represented by a vector “(3,2)” or as “3 red + 2 black”.
Different types of states

- **Initial state**: Initial distribution of tokens.
- **Reachable state**: Reachable from initial state.
- **Final state** (also referred to as “dead states”): No transition is enabled.
- **Home state** (also referred to as home marking): It is always possible to return (i.e., it is reachable from any reachable state).

*How to recognize these states in the reachability graph?*
Properties of Petri nets

• A Petri net is **bounded** iff for every place \( s \), there is a number \( b \geq 0 \) such that for every reachable marking \( M \), the number of tokens in place \( s \) is \( \leq b \).
Not bounded $\Rightarrow$ Infinite Reachability Graph

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Coverability graph

- Similar to reachability graph but unbounded places can hold an “infinite” number of tokens
- Supported by Woped
Properties of Petri nets

• A Petri net is **bounded** iff for every place $s$, there is a number $b \geq 0$ such that for every reachable marking $M$, the number of tokens in place $s$ is $\leq b$

• A Petri net is **live** iff for every reachable marking $M$ and every transition $t$, there is a marking $M'$ reachable from $M$, which enables $t$ (all reachable markings are home markings)
We can make this Petri net live by modifying just one arc. Which one?
Properties of Petri nets

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- A Petri net is **deadlock-free** iff there a reachable marking $M$ that enables no transitions? (i.e. no final states)
Not deadlock-free

- The deadlock can be eliminated by deleting one arc. Which one?
Is this net bounded? live? deadlock-free?
Homework 5: Petri nets

- https://courses.cs.ut.ee/2016/sm/Main/Assessment
- Can be done in teams of up to 4 students

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