Concurrent Systems Modeling using Petri Nets

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(Based on lecture material by Wil van der Aalst
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Limitations of state machines

- Three doctors in a medical centre

- What if there are 6 doctors?
- What if doctors can arrive and leave (so long as they are not busy)?
- State explosion…

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Concurrent systems modeling

• State machines are useful to model behaviour of sequential systems
• But many systems are concurrent by nature
• Statecharts overcome some of the limitations (cf. concurrent regions), but not all
• Petri nets are a family of techniques for modeling systems with concurrency, communication and synchronization
Petri nets

• Simple technique for concurrent systems modeling
  – Three elements: **places**, **transitions** and **arcs**.
  – Graphical and mathematical description.
  – Formal semantics suitable for static analysis.

• Supported by verification and simulation tools
  – ProM, LoLa, PIPE, **Woped**

• Once you understand Petri nets, you will be better equipped to understand other techniques for modeling systems with concurrency, e.g.
  – BPMN
  – UML Activity Diagrams
Rules

• Connections are directed.
• No connections between two places or two transitions.
• Places may hold zero or more tokens.
• First, we consider the case of at most one arc between two nodes.
Enabled Transition

- A transition is **enabled** if each of its input places contains at least one token.
Firing

• An enabled transition can fire (i.e., it occurs).
• When it fires it consumes a token from each input place and produces a token for each output place.
“Token Game”

- In the new state, \textit{make\_picture} is enabled. It will fire, etc.
Remarks

• Firing is **atomic**.
• Multiple transitions may be enabled, but only one fires at a time
• By default, choice is **non-deterministic**
• The **state** of a net is a distribution of tokens over places (also referred to as **marking**).
• Any state machine can be trivially converted into a Petri net – How?
Two transitions are enabled but only one can fire.
Example: Single traffic light
Two traffic lights

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Problem
Solution

How to make them alternate?
Tool

- Woped: A Petri net drawing and animation tool: http://www.woped.org/
Exercise: Doctor’s scenario in Petri nets

Case 1: Patients arrive and leave, number of doctors fixed
Case 2: Patients arrive and leave, doctors arrive and leave (but only leave when they are free)
Case 3: When patients arrive, they are classified into simple and complex cases. Simple cases require only a doctor, complex cases require a doctor and a nurse. (Assume doctors and nurses do not arrive nor leave)
Exercise: Train system

- Consider a circular railroad system with 4 (one-way) tracks (1,2,3,4) and 2 trains (A,B). No two trains should be at the same track at the same time and we do not care about the identities of the two trains.
Multiple arcs connecting two nodes

• The number of arcs between an input place and a transition determines the number of tokens required to be enabled.
• The number of arcs determines the number of tokens to be consumed/produced.
Example: Ball game

Which transition(s) is/are enabled?
Exercise: Burning alcohol.

- Model $\text{C}_2\text{H}_5\text{OH} + 3 \times \text{O}_2 \Rightarrow 2 \times \text{CO}_2 + 3 \times \text{H}_2\text{O}$
- Assume that there are two steps: first each molecule is disassembled into its atoms and then these atoms are assembled into other molecules.
Vending Machine

A vending machine sells chocolate bars. The machine sells small bars for 1.5 euro and large bars for 2 euros. The machine accepts coins of 50 cents, 1 euro and 2 euro. The machine is not able to return coins back to customers. Accordingly, the machine never allows a user to insert more than 2 euros. Once the user has put 2 euros, the machine will not accept any more bills. Instead, it will only allow the user to push the button and get a large bar. If the user has inserted 1.5 euros, he/she may opt to get a small bar, or put an additional 50 cents to get a large bar.
You should be able to ...

• Explain what is a Petri net and what are the basic elements of (plain) Petri nets
• Play a token game on a Petri net.
• Model simple concurrent systems using Petri nets.