# Computer Graphics

Basic II Math Tasks

1. The default cube in Blender has 8 vertices, 6 faces and 12 triangles. How many vertices, vertex normals and faces will it have in the OBJ file format when you triangulate the faces upon export?

2. How many vertices, vertex normals and triangles will the cube from the previous question have when rendering it via the standard graphics pipeline so that it keeps sharp edges?

3. Illustrate the difference between a C^1 and G^1 splines with drawings.

4. Use the learned approach of creating any polynomial curve to find the blending functions of a cubic Bezier curve. The cubic Bezier curve has the derivatives at the endpoints 3 times the difference between the endpoint and the neighboring control point.
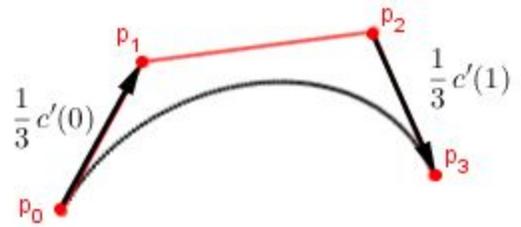
4.1 Write the general parametric cubic curve function and its derivative function.

$$c(x) =$$
$$c'(x) =$$

4.2 Write out the constraints using the found functions and the control points of the Bezier curve.

$$c(0) =$$
$$c'(0) =$$
$$c'(1) =$$
$$c(1) =$$

4.3 Use the found relations and reorder the terms such that the unknowns are on the left hand side and single control points are on the right hand side for each constraint.

4.4. Write out the constraint matrix and unknowns vector from the system of constraints.

$$C = \begin{pmatrix} & & \\ & & \\ & & \\ & & \end{pmatrix} \cdot \begin{pmatrix} \\ \\ \\ \end{pmatrix} = \begin{pmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \end{pmatrix}$$

4.5 Find the blending matrix. Use an online matrix calculator.

$$B = \begin{pmatrix} & & \\ & & \\ & & \\ & & \end{pmatrix}$$

4.6 Write out the blending functions.

$$b_0(t) =$$
$$b_1(t) =$$
$$b_2(t) =$$
$$b_3(t) =$$

4.7 Use algebraic expansions to convert the found functions to the form of known blending functions of Bezier curves.

$$b_0(t) =$$
$$b_1(t) =$$
$$b_2(t) =$$
$$b_3(t) =$$

4.8 How are these known functions called?

5. 1D Perlin noise with linear interpolation and normalized gradient vectors consists of a finite number of shapes. What are they? Draw illustrations of each of them.

6. Find the minimum and maximum values for the 1D Perlin noise described in the previous question.

7. Generalize intuitively the previous answer to 2D, 3D and ND Perlin noise.

**8. Perlin noise will always have a repeating artefact in it, what is it and why does it occur?**

9. In Boids one of the traditional rules has more computational complexity than the others. Which one and what is that complexity in a naive implementation?

10. Derive the quadratic function for solving ray-circle intersection testing. Use the given parametric ray equation and implicit circle equation (*P* is a point on the circle) for deriving it. Assume that the circle is always centered in the origin. Note that the square (*n* = 2) polynomial expansion works for vectors if we use the dot product for the multiplication operation.
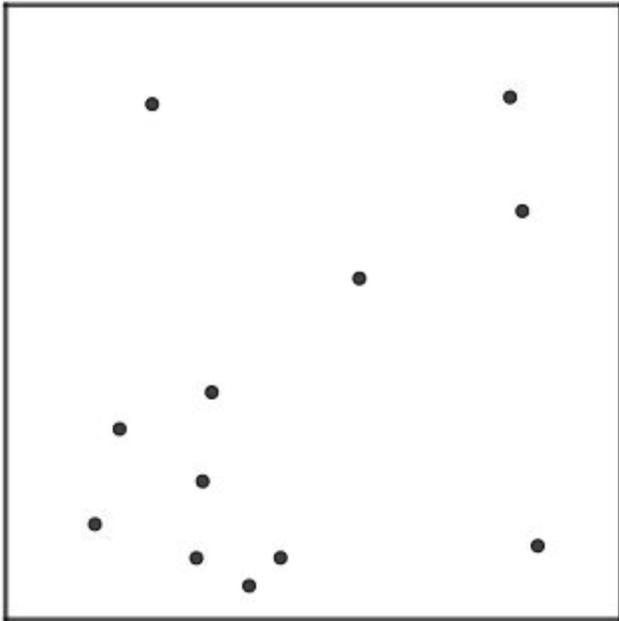
$$Ray(t) = S + tD$$
$$Circle : P^2 = r^2$$

11. Use the previously derived quadratic equation to find the values of *t* for the intersections between the following rays and the circle:
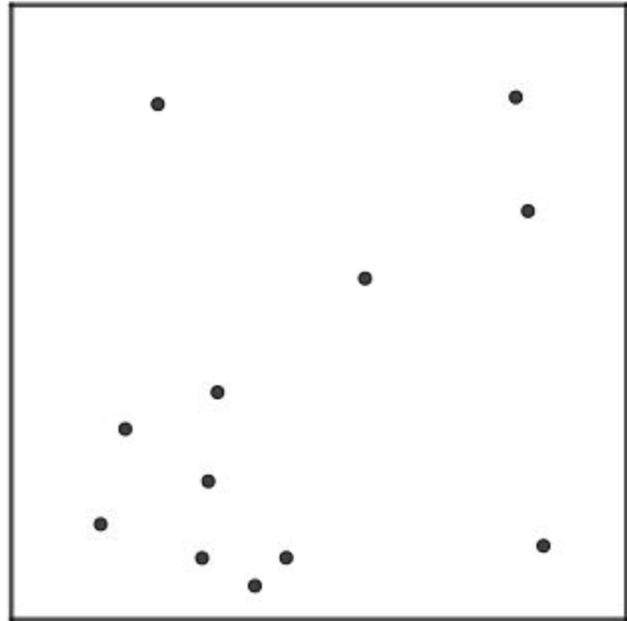

$S_1 = (-4, 3)$

$S_2 = (-1, 3)$

$S_3 = (2, 3)$

$D_2 = (0, -0.75)$

$D_1 = (1, -0.75)$

$D_3 = (-1, -0.75)$

$r = 1$
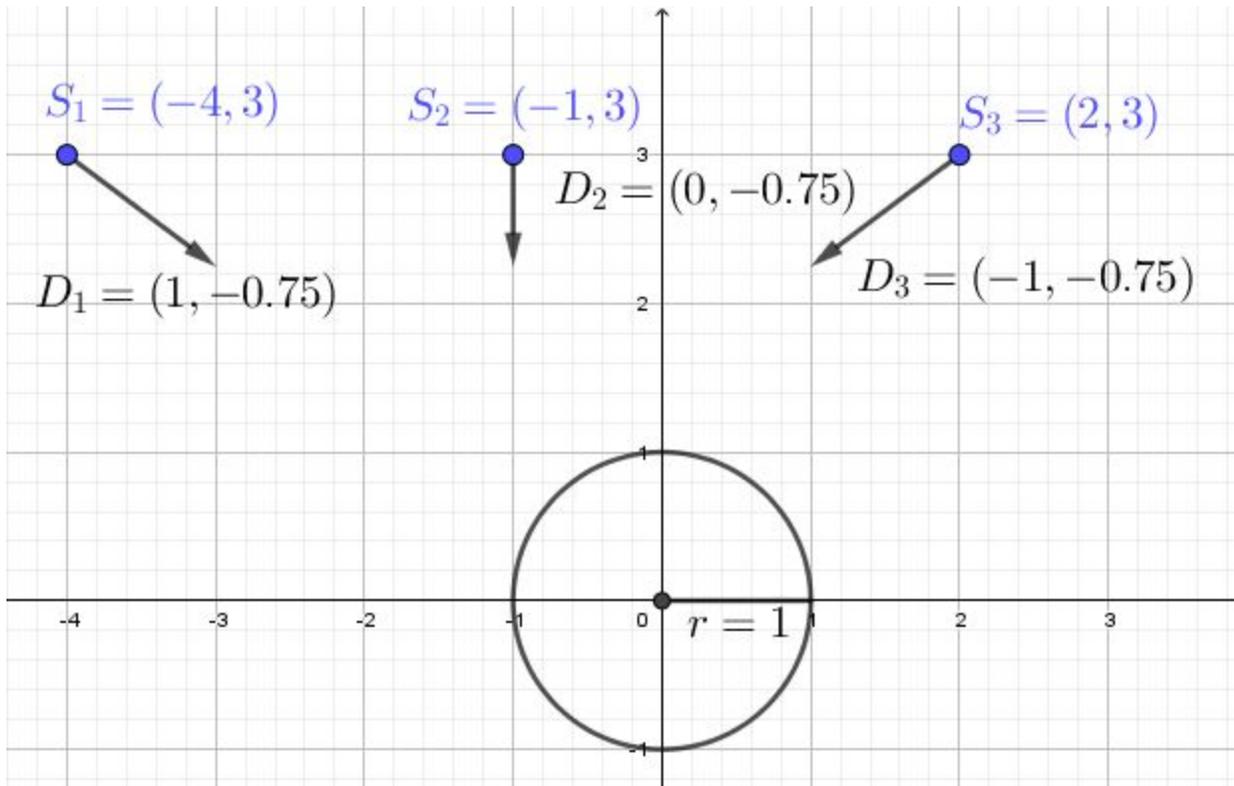
12. How would you extend the solution for finding intersections with circles not in the origin?

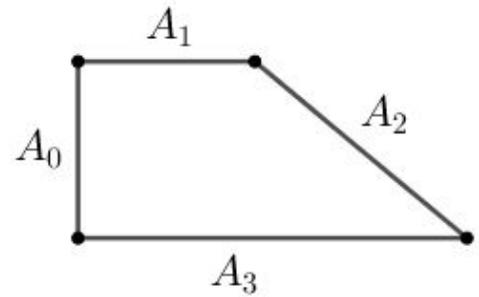**13. Draw the two space partitioning trees:**

Quadtree with max cell size 2

Standard K-D tree

14. Consider the radiosity algorithm with the following view factors and geometry:

$$F = \begin{pmatrix} 0 & 0.30 & 0.30 & 0.40 \\ 0.30 & 0 & 0.50 & 0.20 \\ 0.20 & 0.05 & 0 & 0.75 \\ 0.15 & 0.30 & 0.55 & 0 \end{pmatrix}$$

Make the patch A_1 emit 100 units of light. Create a script to find the approximations of the outputted light using the Jacobi iteration method. Start the approximation with a 0-vector. Consider only a single color channel and the following 3 cases for the surface diffuse scattering coefficients. Round the result values to the nearest integer.

Case 1:

$$\rho = (0.7, 0, 0.7, 0.7)$$
$$L^4 = (\qquad , \qquad , \qquad)$$
$$L^8 = (\qquad , \qquad , \qquad)$$

Case 2:

$$\rho = (0.2, 0, 0.7, 0.7)$$
$$L^4 = (\qquad , \qquad , \qquad)$$
$$L^8 = (\qquad , \qquad , \qquad)$$

Case 3:

$$\rho = (0.7, 0, 0.2, 0.7)$$
$$L^4 = (\qquad , \qquad , \qquad)$$
$$L^8 = (\qquad , \qquad , \qquad)$$

15. You are given a sphere map, which is equivalent to if a reflective sphere has been rendered with an orthographic camera with the view matrix V_{ort} from the top. Your goal is to do environment mapping on the scene objects.

15.1 Given a surface point *p* with a normal *n*, from what coordinates should you sample there sphere map from?

Orthographic camera
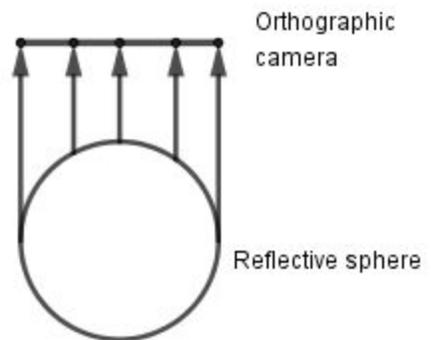
Reflective sphere

15.2 Your goal is now to only use the sphere map for reflecting the static sky. Assume that the reflective sphere was relatively close to the ground. How do you change the sphere map sampling?

16. Compare the memory consumption of a regular shadow map and cascaded shadow mapping. Assume power-of-two textures. Perspective projection FOV_h is 80°, near plane is at 1, far plane is at 300. Light is coming straight down from a directional light source above the scene. Cascade borders are at 100 and 200. Each cascade has 2 times lower resolution, starting from 1024×1024. Regular shadow map has resolution 2048×2048.