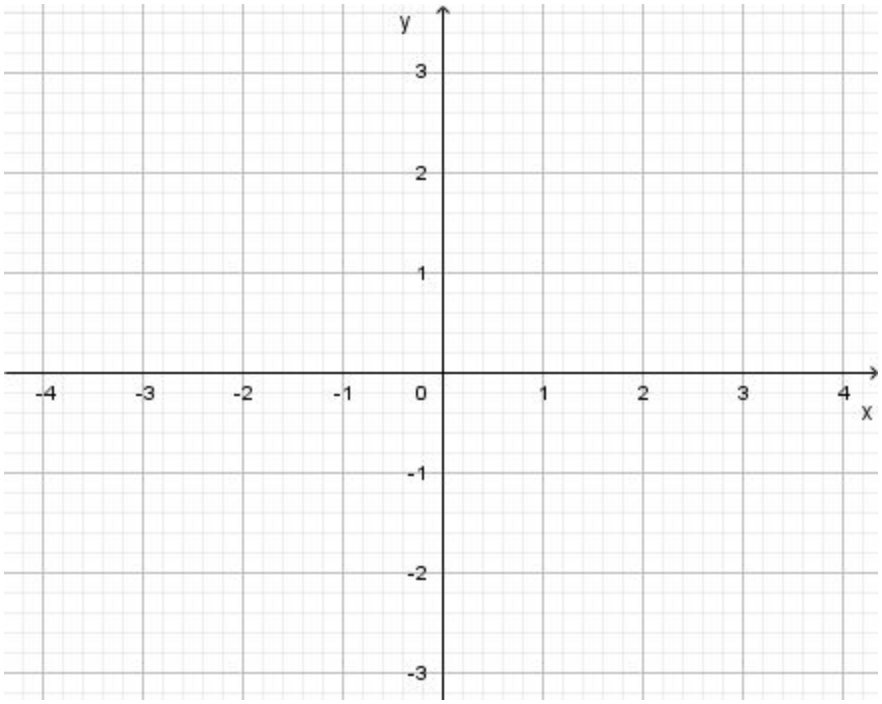
Computer Graphics

Basic Preliminary Math Tasks

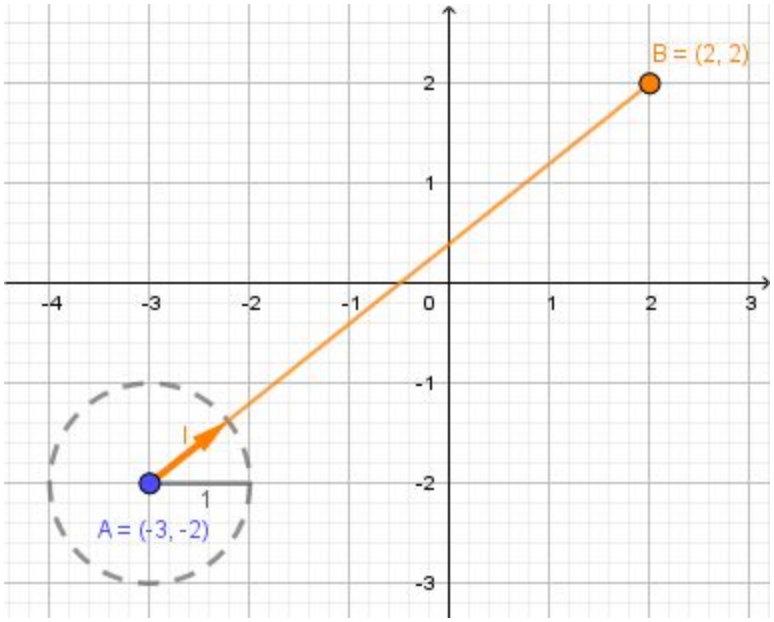
1. Find a vector v from point $A = (-4,\ 2)$ to point $B = (2,\ 1)$.

2. Draw on the graph the points A, B and vector v from the previous task. Write the coordinates on each of the objects on the graph.

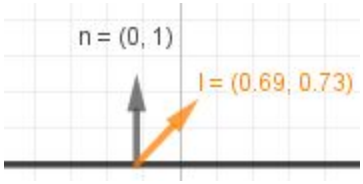


3. Find the length of and normalize a vector $n = (3,\ 4,\ 0)$.

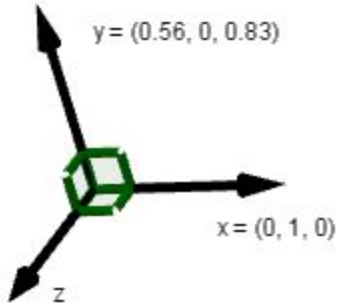
4. Calculate the approximate coordinates of the vector l on the graph below.



5. Find the cosine of the angle between vectors $n = (0,\ 1)$ and $l = (0.69,\ 0.73)$ using the dot product operation.

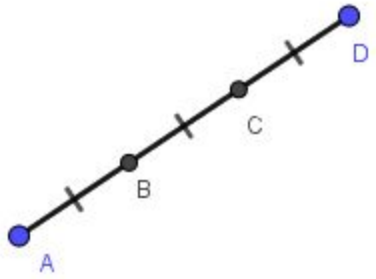


6. Find the coordinates of vector z orthogonal to vectors x and y (all depicted on the graph below) using the cross product operation.



7. What is the handedness of the basis from the previous task?

8. The points B and C divide the line segment from point A to point D into three equal parts. Express the points B and C in terms of the points A and D.



9. Calculate the coordinates of the points B and C from the previous task if $A = (1, 3)$ and $D = (4,\ 5)$.

10. Perform the following matrix-vector multiplication:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \\ 5 \end{pmatrix} =$$

11. Come up with a 3x3 matrix that multiplies the first coordinate of a 3-dimensional vector by 4, the second coordinate by 1 and the third coordinate by 2.

12. Transpose the following orthonormal matrix:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -0.6 & -0.8 \\ 0 & 0.8 & -0.6 \end{pmatrix}^{\mathrm{T}} =$$

13. Perform the following matrix-vector multiplication:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -0.6 & -0.8 \\ 0 & 0.8 & -0.6 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix} =$$

14. Use the result from the task 13. as a vector v and the result from task 12. as a matrix M and calculate the matrix-vector multiplication $M \cdot v$.

15. What should the values for s, t and u be in the following matrix in order to add 2 to the first coordinate and 5 to the second coordinate of the vector $(x, y, 1)$?

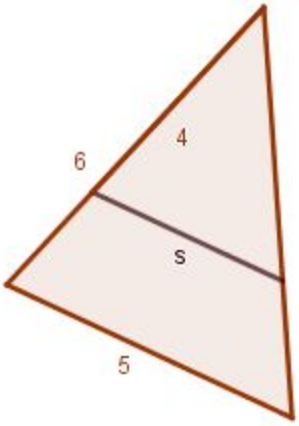
$$\begin{pmatrix} 1 & 0 & s \\ 0 & u & t \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} x+2 \\ y+5 \\ 1 \end{pmatrix}$$

16*. Perform the matrix-vector multiplication on the standard basis vectors. Where in the original matrix can you find the results? What useful conclusion can you draw from that in terms of the entire vector space transformation?

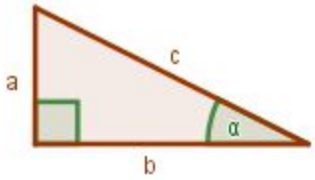
$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} =$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} =$$

17. Find the length s in the triangle below.



18. Define the cosine, sine and tangent functions as ratios of sides of a right triangle.



19. Draw the cosine and sine functions to the graphs below.