LTAT.05.008: Software Analytics

Experiments & Statistical Process Control (SPC)

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Quiz 3: Case Studies

(10 min -- 2 marks)
Data Collection & Research Methods in Data Science

• Survey
  – Questionnaire-based (primary study)
  – Literature-based (secondary / tertiary study)

• Case Study
  – Exploratory
  – Descriptive
  – Confirmatory/Explanatory
  – Improving

• Experiment
  – Controlled Experiment
  – Quasi-Experiment

• Many other...
  – Action Research
  – Ethnography
  – Longitudinal Studies
  – Design Science
Structure of Lecture 4

• Experiments
• Exercise / Homework 3
• Statistical Process Control (SPC)
Descriptive vs. Inferential Statistics

- **Descriptive statistics**
  - characterizes data
  - usually quantitative data
  - data may be drawn from the whole population or just be a sample

- **Inferential statistics**
  - intends to characterize a population by inferring from sample data
  - essential prerequisite: random sampling
Experimental Designs: Random sampling & assignment

Randomization is a prerequisite for a controlled experiment!

- **is random assignment used?**
  - yes
    - randomized or true experiment
  - no
    - is there a control group or multiple measures?
      - yes
        - quasi-experiment
      - no
        - non-experiment
Experimental Designs

- **One-Group designs (within-group):**
  - Post-Test
    \[ X \ O \]
  - Pre-Test and Post-Test
    \[ O \ X \ O \]
  - Interrupted time-series
    \[ O \ O \ X \ O \ O \ O \ X \ O \ X \ O \ ... \]

  With:
  \[ O = \text{observation (measurement)} \]
  \[ X = \text{treatment (intervention)} \]

- **Multiple-Group designs (between-groups):**
  - With or without random sampling / assignment
  - With or without blocking
  - Balanced or unbalanced
  - Factorial Designs:
    - nested vs. crossed
    - interaction between factors
Experimental Designs – 2 Groups Example

Group = Set of “experimental units” (subjects)

Vertical alignment of Os shows that pretest and posttest are measured at same time

R indicates the groups are randomly assigned

Os indicate different waves of measurement

There are two lines, one for each group

X is the treatment

Time

http://www.socialresearchmethods.net/kb/design.php
Experimental Designs: Randomization

- **Definition [Pfl94]:**
  - Randomization is the random assignment of subjects to groups or of treatments to experimental units, so that we can assume independence (and thus validity) of results.

- **Rationale for Randomization [Pfl94]:**
  - Sometimes the results of an experimental treatment can be affected by the time, the place or unknown characteristics of the participants (= experimental units / subjects)
  - These uncontrollable factors can have effects that hide or skew the results of the controllable variables.
  - To spread and diffuse the effects of these uncontrollable or unknown factors, you can
    - assign the order of treatments randomly,
    - assign the participants to each treatment randomly, or
    - assign the location of each treatment randomly, whenever possible.
Experimental Designs: Blocking /1

• **Definition [Pfl94]:**
  – Blocking (Stratification) means allocating experimental units to blocks (strata) or groups so the units within a block are relatively homogeneous.

• **Rationale for Blocking [Pfl94]:**
  – The blocked design captures the anticipated variation in the blocks by grouping like varieties, so that the variation does not contribute to the experimental error.
Experimental Designs: Blocking /2

- **Example [Pfl94]:**
  - Suppose you are investigating the comparative effects of two design techniques A and B on the quality of the resulting code.
  - The experiment involves teaching the techniques to twelve developers and measuring the number of defects found per thousand lines of code to assess the code quality.
  - It may be the case that the twelve developers graduated from three universities. It is possible that the universities trained the developers in very different ways, so that the effect of being from a particular university can affect the way in which the design technique is understood or used.
  - To eliminate this possibility, three blocks can be defined so that the first block contains all developers from university X, the second block from university Y, and the third block from university Z. Then, the treatments are assigned at random to the developers from each block. If the first block has six developers, you would expect three to be assigned to design method A, and three to method B, for instance.
Experimental Designs: Blocking /3

without blocking

with blocking

Less variance increases statistical power (for the same mean difference)
Experimental Designs: Balancing

- **Definition [Pfl94]:**
  - Balancing is the blocking and assigning of treatments so that an equal number of subjects is assigned to each treatment, wherever possible.

- **Rationale for Balancing [Pfl94]:**
  - Balancing is desirable because it simplifies the statistical analysis, but it is not necessary.
  - Designs can range from being completely balanced to having little or no balance.
(Simple) Between-Subject Design

- Treatment (Intervention):  
  - 1 Factor, e.g., Tool  
  - 2 Levels, e.g., Tool T1 and Tool T2

- Between-subject design:  
  - Group A uses T1  
  - Group B uses T2
(Simple) Within-Subject Design

- **Treatment (Intervention):**
  - 1 Factor, e.g., Tool
  - 2 Levels, e.g., Tool T1 and Tool T2

- **Within-subject design:**
  - Random sample uses both T1 and T2 (in a certain order)
(Simple) Within-Subject Design

- Treatment (Intervention):
  - 1 Factor, e.g., Tool
  - 2 Levels, e.g., Tool T1 and Tool T2

- Within-subject design:
  - Random sample uses both T1 and T2 (in a certain order)

Question: what if order influences the outcome?
(Simple) Within-Subject Design

- **Treatment (Intervention):**
  - 1 Factor, e.g., Tool
  - 2 Levels, e.g., Tool T1 and Tool T2

- **Within-subject design:**
  - Random sample uses both T1 and T2 (in a certain order)

- **Within-subject design with counter-balancing (if order could play a role):**
  - Group A uses first T1 then T2
  - Group B uses first T2 then T1
Interaction Effects

- **Example**: Measuring time to code a program module with or without using a reusable repository
  - **Case 1**: large difference
  - **Case 2**: small difference

![Diagram showing time to code with and without reuse for Case 1 and Case 2]
Interaction Effects

- **Example:** Measuring time to code a program module with or without using a reusable repository
  - **Case 1:** No interaction between factors
  - **Case 2:** Interaction effect → Effect on Time to Code (Factor 1) depends (also) on Size of Module (Factor 2)

Case 1

Case 2
Interaction Effects ➔ Factorial Designs

- The previous slide motivates why factorial designs can be necessary:
  - While the original question is (only) whether reuse changes coding time, it might turn out that there is an interaction with some other (yet uncontrolled factor).
    - In the example, this uncontrolled factor is 'size of module' (in the case on the right hand side).

- Thus, we need a more complex design with two factors: reuse & module size ➔ factorial design (see next slides)
Experimental Designs: Factorial Designs

- **Definition of “Factorial Design”:**
  - The design of an experiment can be expressed by explicitly stating the number of factors and how they relate to the different treatments.
  - Expressing the design in terms of factors, tells you how many different treatment combinations are required.

- **Crossed (Cross-over) Design:**
  - Two factors, F1 and F2, in a design are said to be crossed if each level of each factor appears with each level of the other factor.

- **Nested Design:**
  - Factor F2 is nested within factor F1 if each meaningful level of F2 occurs in conjunction with only one level of factor F1.
Experimental Designs: Crossed vs. Nested

- **Factorial Design:**
  - **Crossing** (each level of each factor appears with each level of the other factor)
  - **Nesting** (each level of one factor occurs entirely in conjunction with one level of another factor)

- Proper nested or crossed design may reduce the number of cases to be tested.

The difference lies in the way how the statistical analysis (ANOVA) is conducted; details can be found here:

[https://onlinecourses.science.psu.edu/stat502/node/152](https://onlinecourses.science.psu.edu/stat502/node/152)
Experimental Designs: Crossed vs. Nested

Factorial Design:

- **Crossing** (each level of each factor appears with each level of the other factor)
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Useful for looking at two factors, each with two or more conditions.

Useful for investigating one factor with two or more conditions.

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Mixed Design

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Experimental Designs: Crossed vs. Nested

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Experimental Designs: Design Selection

Flow Chart for selecting an Experimental Design [Pfl95]

- Also appeared as: S. L. Pfleeger: Experimental design and analysis in software engineering, Parts 1 to 5, Software Engineering Notes, 1995 and 1996.
Selecting the Research Method

- The choice of method depends among other things on:
  - Suitable study subject (e.g., do participants have enough experience?)
  - Possibility to control the environment
  - The size/scale/cost of the study
  - The need for generality in the results
  - Availability of information/data and other resources
  - What is the purpose of the study? (exploration, prediction, understanding of cause-effect relations, applicability of results in industry, ....)

- Difficult to provide general recommendation with respect to choice of method
Experiment or Case Study or Survey?

- **(Controlled) Experiments** give the researchers freedom to isolate a defined effect and to hold other things constant (-> research in-depth)
  - Often difficult to avoid that people respond differently than they would have done in a natural environment
  - Also, you might have a too restricted setting and omit important influencing factors that play a role in a natural environment

- **Case Studies** have the advantage that you observe people doing what they are actually doing in their natural environment (-> research in-the-typical)
  - It is limited what you can control without interfering with natural activity

- **Surveys** provide researchers with information about what many people (think/report they) are doing (-> research in-breadth)
  - No control whatsoever (issues relate to data validity and representativeness)
Validity & Reliability of Empirical Studies

• Construct Validity
  – Concepts being studied are operationalised and measured correctly (do the measures used actually represent the concepts you want to measure?)

• Internal Validity
  – Establish a causal relationship and sort out spurious relationships (exclude confounding variables / by: random sampling, blocking, balancing)

• Conclusion Validity
  – Do proper statistical inference

• External Validity
  – Establish the domain to which a study’s findings can be generalized (precisely describe the population and experimental conditions)

• Reliability
  – The study can be repeated (i.e., by other researchers) and yields the same results
  – The measurement instrument is reliable (interrater agreement)
Beware of the following "effects":

- **Hawthorne-effect**
  - The attention given to the workers in "Hawthorne Plant of the Western Electric Company" is thought to have had greater effect than the process changes that were studied.

- **Observer effect (= Heisenberg-effect)**
  - Refers to changes that the act of observing (measurement) will make on the phenomenon being observed. (somewhat similar to Hawthorne-effect)

- **Weinberg-effect**
  - Study showed that system developers who knew that they were being assessed, decreased their performance on most parameters which were not assessed.

- **Observer-expectancy effect (theory-loaded observation/cognitive bias)**
  - What you (think you) observe, is influenced by what you think you will observe.

- **Subject-expectancy effect (cognitive bias)**
  - A cognitive bias that occurs when a subject expects a given result and therefore unconsciously manipulates an experiment or reports the expected result.

- **The effect of the question’s structure (construction, forming)**
  - “Do you think the lecture today was A: good, B: very good, C: brilliant, D: exceptional"
Literature on (controlled) experiments in SE

• S. L. Pfleeger (1995-96) “Experimental design and analysis in software engineering”, Parts 1 to 5, Software Engineering Notes
Structure of Lecture 4

- Experiments
- Exercise / Homework 3
- Statistical Process Control (SPC)
Homework 3

• Task 1 [5 marks]
  – Review a reported SE experiment
  – Use Checklist

• Task 2 [5 marks]
  – Statistical Process Control
Homework 3: Task 1 – Checklist

• Page 1

Checklist for Assessing the Quality of Papers Reporting Controlled Experiments in Software Engineering Research

<table>
<thead>
<tr>
<th>Question</th>
<th>How to answer the question</th>
<th>Answer</th>
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</thead>
<tbody>
<tr>
<td>Category: Questions on Aims</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Do the authors clearly state the aims of the research?</td>
<td>Consider: • Do the authors state research questions, e.g., related to time-to-market, cost, product quality, process quality, developer productivity, developer skill? • Do the authors state hypotheses and their underlying theories?</td>
<td>To full extent (fully) O</td>
</tr>
<tr>
<td>Category: Questions on Design, Data Collection, and Data Analysis</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. Do the authors</td>
<td>Consider:</td>
<td></td>
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</table>

Question to be answered
Answer choices (check one!)
Remember to provide a justification for your choices!
just list of hints .... (incomplete lists!)
Exercise: Assessing the Quality of Reported Experiments

- Paper
- Checklist
  - Focus on items 6 & 7
- Work individually
  - Make sure you take notes on the rationale for your assessment
- After ca. 10 min compare with your neighbour
- Report to class
Structure of Lecture 4

• Experiments
• Exercise / Homework 3
• Statistical Process Control (SPC)
Basics of Statistical Process Control

- **Statistical Process Control (SPC)**
  - monitoring production process to detect and prevent poor quality

- **Sample**
  - subset of items produced used for inspection

- **Control Charts**
  - process is within statistical control limits
Variability

• Random
  – common causes
  – inherent in a process
  – can be eliminated only through improvements in the system

• Non-Random
  – special causes
  – due to identifiable factors
  – can be modified through operator or management action
Statistical Process Control

- Understanding the process,
- Understanding the causes of variation, and
- Elimination of the sources of special cause variation.
Usage of control charts

1. Select process
2. Identify product or process characteristics that describe process performance
3. Select the appropriate type of control chart
4. Measure process performance over a period of time
5. Use appropriate calculations based on measurement data to determine center lines and control limits for performance characteristics
6. Plot measurement data on control charts
7. Are all measured values within limits and distributed randomly around centerlines?
8. Process is stable; continue measuring
9. Process is not stable
10. Identify and remove assignable causes

Source: Florac & Carleton (1999)
Process Control Chart

Upper control limit

Process average

Lower control limit

Out of control

Sample number

1 2 3 4 5 6 7 8 9 10
Normal Distribution

Choices of Control Limits

-3σ \(-2σ\) \(-1σ\) \(μ=0\) 1σ 2σ 3σ

95% 99.74%
A process is in control if …

1. … no sample points outside limits
2. … most points near process average
3. … about equal number of points above and below centerline
4. … points appear randomly distributed
A process is out of control (or has suspicious patterns) if …

- 8 consecutive points on one side of the center line
- 1 single point outside zone C
- 2 out of 3 consecutive points in zone C but still inside the control limits
- 4 out of 5 consecutive points in zone B
Detecting out-of-control situations

Test 4:
- 8 successive points on same side of centerline

Test 3:
- 4 out of 5 points in zone B

Test 2:
- 2 out of 3 points beyond zone B

Test 1:
- Single point outside zone C

Source: Western Electric (1958)
Common questions for investigating an out-of-control process (1)

• Are there differences in the measurement accuracy of instruments/methods used?
• Are there differences in the methods used by different personnel?
• Is the process affected by the environment?
• Has there been a significant change in the environment?
• Is the process affected by predictable conditions?
  – Example: tool wear.
• Were any untrained personnel involved in the process at the time?
Common questions for investigating an out-of-control process (2)

• Has there been a change in the source for input to the process?
  – Example: plans, specs, information.
• Is the process affected by employee fatigue?
• Has there been a change in policies or procedures?
  – Example: maintenance procedures.
• Is the process adjusted frequently?
• Did the samples come from different parts of the process? Shifts? Individuals?
• Are employees afraid to report “bad news”?

One should treat each “Yes” answer as a potential source of a special cause.
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Source: Florac & Carleton (1999)
Ishikawa Chart Example: Change Request Process

Collection
- Problem reports not logged in properly
- Information missing from problem reports
- Problem cannot be replicated

Evaluation
- Cannot isolate software artifact(s) containing the problem
- Cannot determine what needs to be done to fix the problem

Resolution
- Change control board meets only once a week
- Change decisions not released in a timely manner
- Delays in approving changes
- Takes time to make changes
- Must reconfigure baselines

Closure
- Delays in shipping changes and releases
- Delays en-route
- It takes too long to process software change requests

- Software Analytics © Dietmar Pfahl 2018
Usage of control charts

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Source: Florac & Carleton (1999)
Type of Chart depends on Type of Measures

• **Attribute**
  – a product characteristic that can be evaluated with a discrete response
    • good - bad; yes – no
    • 0, 1, 2, 3, ... issues

• **Variable**
  – a product characteristic that is continuous and can be measured
    • e.g., complexity, length, rates

• **Types of charts**
  – **Attributes**
    • p-chart / np-chart
    • c-chart / u-chart
  – **Variables**
    • 1-MR chart (MR: moving range)
    • X-bar R-chart (means & range)
    • X-bar S-chart (means & std. dev.)
Type of Chart depends on Type of Measures

- **Binomial Distrib.**
  - Item: good/bad
  - bad = ‘defective’

- **Poisson Distrib.**
  - Item: 0, 1, 2, 3 ... defects

- **np chart**
- **u chart**
- **c chart**
- **\( \bar{X} - R \) chart**
- **\( \bar{X} - S \) chart**
Control Charts for Attributes

- **p-Chart**
  - Controls ‘portion defective’ in a sample
- **np-Chart**
  - Controls ‘number of defectives’ in a sample
- **u-Chart**
  - Controls ‘number of defects per size unit’ in samples of varying size
- **c-Chart**
  - Controls ‘number of defects’ in samples of size 1 (items)
p-Chart

\[
UCL = \bar{p} + z\sigma_p
\]

\[
LCL = \bar{p} - z\sigma_p
\]

\[
\bar{p} \pm 3\sqrt{\frac{\bar{p}(1 - \bar{p})}{n}}
\]

\[z = \text{number of standard deviations from process average}\]

\[\bar{p} = \text{sample proportion defective; an estimate of process average}\]

\[\sigma_p = \text{standard deviation of sample proportion}\]

\[n = \text{sample size}\]

\[m = \text{number of samples}\]
p-Chart Example

<table>
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<tr>
<th>SAMPLE</th>
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<th>PROPORTION DEFECTIVE</th>
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<td>.06</td>
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Note: This example uses constant sample sizes $n$.

There exist versions of the p-Chart for varying sample sizes.
p-Chart Example (cont.)

\[
p = \frac{\text{total defectives}}{\text{total sample observations}} = \frac{200}{20 \times 100} = 0.10
\]

\[
\text{UCL} = p^+ z \sqrt{\frac{\bar{p}(1 - \bar{p})}{n}} = 0.10 + 3 \sqrt{\frac{0.10(1 - 0.10)}{100}}
\]

UCL = 0.190

\[
\text{LCL} = p^- z \sqrt{\frac{\bar{p}(1 - \bar{p})}{n}} = 0.10 - 3 \sqrt{\frac{0.10(1 - 0.10)}{100}}
\]

LCL = 0.010
p-Chart
Example (cont.)
np-Chart

• An adaptation of the p-chart
  – Used in situations where personnel find it easier to interpret process performance in terms of concrete numbers of units rather than the somewhat more abstract proportion.

• The np-chart differs from the p-chart in the following aspects:
  – Control limits:
    \[ n\bar{p} \pm 3\sqrt{n\bar{p}(1 - \bar{p})} \]
  – The number non-conforming items (np), rather than the fraction nonconforming (p), is plotted against the control limits.
  – The sample size n must be constant.
c-Chart

\[
\begin{align*}
\text{UCL} &= \bar{c} + 3\sigma_c \\
\text{LCL} &= \bar{c} - 3\sigma_c \\
\sigma_c &= \sqrt{\bar{c}}
\end{align*}
\]

Where

\( c \) = number of defects per sample (e.g., code tested per day)
### c-Chart (cont.)

Number of defects in 15 days

<table>
<thead>
<tr>
<th>SAMPLE</th>
<th>NUMBER OF DEFECTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>16</td>
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</tr>
<tr>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td><strong>190</strong></td>
</tr>
</tbody>
</table>

\[
\bar{c} = \frac{190}{15} = 12.67
\]

**UCL**
\[
= \bar{c} + z\sigma_c
= 12.67 + 3\sqrt{12.67}
= 23.35
\]

**LCL**
\[
= \bar{c} + z\sigma_c
= 12.67 - 3\sqrt{12.67}
= 1.99
\]
c-Chart
(cont.)
c-Chart vs. u-Chart

\[ \text{UCL} = \bar{c} + 3\sigma_c \]
\[ \text{LCL} = \bar{c} - 3\sigma_c \]

where

\[ c = \text{number of defects per sample (e.g., code tested per day)} \]

\[ \sigma_c = \sqrt{\bar{c}} \]

Note: If data is collected for sample size >1 (e.g., using LOC of the code tested/inspected per day as sample), then the u-Chart should be used (allows for varying sample size n). Plot/control defect density (d/n) instead of defects (d).
Type of Chart depends on Type of Measures

Process characteristic or measurement -> What type of data?

- ATTRIBUTE
- CONTINUOUS

What type of attribute data?

DEFECTIVES
- p chart
- np chart

DEFECTS
- u chart
- c chart

What subgroup size?

- 1
- 2 – 10
- > 10

Binomial Distrib.

Poisson Distrib.

Item: good/bad
bad = ‘defective’

Item: 0, 1, 2, 3 ... defects
Control Charts for Variables

- Mean chart (x-bar-Chart)
  - plots means of samples (of constant size)
- Range chart (R-Chart or S-Chart)
  - plots amounts of dispersion in a sample
    - Range for sample size <10
    - Std. Deviation for larger sample sizes
- Used to monitor variables data when samples are collected at regular intervals from a business or industrial process where normal distribution of the data can be assumed
Using x-bar and R-Chart together

Process average and process variability must be in control.

- First, variance is checked with R-Chart
- If the R-Chart indicates that the variance is out-of-control, it doesn’t make sense to check the mean value (x-bar Chart)
## Example Data

### Observations (Slip-Ring Diameter, CM)

<table>
<thead>
<tr>
<th>SAMPLE k</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>( \bar{x} )</th>
<th>R</th>
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<tr>
<td>1</td>
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<td>4.99</td>
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<td>5.07</td>
<td>4.99</td>
<td>5.03</td>
<td>0.10</td>
</tr>
</tbody>
</table>

**Summary:**

- Mean: 50.09
- Range: 1.15
R-Chart Example (cont.)

\[
\bar{R} = \frac{R}{k} = \frac{1.15}{10} = 0.115 \\
UCL = D_4 \bar{R} = 2.11(0.115) = 0.243 \\
LCL = D_3 \bar{R} = 0(0.115) = 0
\]

Retrieve Factor Values $D_3$ and $D_4$ from Table
<table>
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<tr>
<th>Sample Size = m</th>
<th>A₂</th>
<th>A₃</th>
<th>d₂</th>
<th>D₃</th>
<th>D₄</th>
<th>B₃</th>
<th>B₄</th>
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<tbody>
<tr>
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<td>1.128</td>
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<td>3.267</td>
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<td>2.568</td>
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<td>0.459</td>
<td>1.541</td>
<td>0.565</td>
<td>1.435</td>
</tr>
</tbody>
</table>
R-Chart Example

\[ R_i = \max(x_j) - \min(x_j) \]

\[ R = \frac{\sum_{i=1}^{m} \max(x_{ij}) - \min(x_{ij})}{m} \]

UCL = 0.243
LCL = 0

Sample number

Range
x-bar Chart Example

\[ \bar{x} = \frac{\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij}}{mn} \]

\[ \bar{x}_i = \frac{\sum_{j=1}^{n} x_{ij}}{n} \]

UCL = 5.08

LCL = 4.94

\( \bar{x} = 5.009 \)
x-bar Chart Example (cont.)

\[ \bar{x} = \frac{\bar{x}}{k} = \frac{50.09}{10} = 5.01 \text{ cm} \]

\[ \text{UCL} = \bar{x} + A_2 \bar{R} = 5.01 + (0.58)(0.115) = 5.08 \]

\[ \text{LCL} = \bar{x} - A_2 \bar{R} = 5.01 - (0.58)(0.115) = 4.94 \]

Retrieve Factor Value \( A_2 \)
<table>
<thead>
<tr>
<th>Sample Size = m</th>
<th>A&lt;sub&gt;2&lt;/sub&gt;</th>
<th>A&lt;sub&gt;3&lt;/sub&gt;</th>
<th>d&lt;sub&gt;2&lt;/sub&gt;</th>
<th>D&lt;sub&gt;3&lt;/sub&gt;</th>
<th>D&lt;sub&gt;4&lt;/sub&gt;</th>
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</table>
Homework 2: Presentations
Homework 2: Solution
Homework 3: Experiments and SPC